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Selina ICSE Solutions for Class 9 Maths Chapter 14 Rectilinear Figures [Quadrilaterals: Parallelogram, Rectangle, Rhombus, Square and Trapezium]

Exercise 14(A)

Solution 1:

The sum of the interior angle=4 times the sum of the exterior angles.

Therefore the sum of the interior angles = $4\times360^\circ = 1440^\circ$.

Now we have

$$(2n-4)\times 90^0 = 1440^0$$
$$2n-4 = 16$$
$$2n = 20$$
$$n = 10$$

Thus the number of sides in the polygon is 10.

Solution 2:

Let the angles of the pentagon are 4x, 8x, 6x, 4x and 5x.

Thus we can write

$$4x+8x+6x+4x+5x = 540^0$$
$$27x = 540^0$$
$$x = 20^0$$

Hence the angles of the pentagon are:

$4\times20^\circ = 80^\circ$, $8\times20^\circ = 160^\circ$, $6\times20^\circ = 120^\circ$, $4\times20^\circ = 80^\circ$, $5\times20^\circ = 100^\circ$

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Solution 3:

Let the measure of each equal angles are x.

Then we can write

$$140^0 + 5x = (2\times 6 - 4)\times 90^0$$
$$140^0 + 5x = 720^0$$
$$5x = 580^0$$
$$x = 116^0$$

Therefore the measure of each equal angles are 116°

Solution 4:

Let the number of sides of the polygon is *n* and there are k angles with measure 195°.

Therefore we can write:

$$5\times 90^0 + k\times 195^0 = (2n - 4)90^0$$
$$180^0 n - 195^0 k = 450^0 - 360^0$$
$$180^0 n - 195^0 k = 90^0$$
$$12n - 13k = 6$$

In this linear equation n and k must be integer. Therefore to satisfy this equation the minimum value of k must be 6 to get n as integer.
Hence the number of sides are: 5 + 6 = 11.

Solution 5:

Let the measure of each equal angles are x.

Then we can write:

$$3\times 132^0 + 4x = (2\times 7 - 4)90^0$$
$$4x = 900^0 - 396$$
$$4x = 504$$
$$x = 126^0$$

Thus the measure of each equal angles are 126°.

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Solution 6:

Let the measure of each equal sides of the polygon is x.

Then we can write:

$$142^0 + 176^0 + 6x = (2\times 8 - 4)90^0$$
$$6x = 1080^0 - 318^0$$
$$6x = 762^0$$
$$x = 127^0$$

Thus the measure of each equal angles are 127^{o}.

Solution 7:

Let the measure of the angles are 3x, 4x and 5x.

Thus

$$\angle A + \angle B + \angle C + \angle D + \angle E = 540^0$$
$$3x + (\angle B + \angle C) + 4x + 5x = 540^0$$
$$12x + 180^0 = 540^0$$
$$12x = 360^0$$
$$x = 30^0$$

Thus the measure of angle E will be $4\times 30^{\circ} = 120^{\circ}$

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Solution 8:

(i)

Let each angle of measure x degree.

Therefore measure of each angle will be:

$$x = 180^0 - 2\times 15^0 = 150^0$$

(ii)

Let each angle of measure x degree.

Therefore measure of each exterior angle will be:

$$\begin{aligned} x &= 180^0 - 150^0 \\ &= 30^0 \end{aligned}$$

(iii)

Let the number of each sides is n.

Now we can write

$$\begin{aligned} n\cdot 150^0 &= (2n-4)\times 90^0 \\ 180^0 n - 150^0 n &= 360^0 \\ 30^0 n &= 360^0 \\ n &= 12 \end{aligned}$$

Thus the number of sides are 12.

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Solution 9:

Let measure of each interior and exterior angles are 3k and 2k.

Let number of sides of the polygon is n.

Now we can write:

$$n \cdot 3k = (2n-4) \times 90^0$$
$$3nk = (2n-4)90^0 \qquad \ldots(1)$$

Again

$$n \cdot 2k = 360^0$$
$$nk = 180^0$$

From (1)

$$3 \cdot 180^0 = (2n-4)90^0$$
$$3 = n-2$$
$$n = 5$$

Thus the number of sides of the polygon is 5.

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Solution 10:

For (n-1) sided regular polygon:

Let measure of each angle is x.

Therefore

$$(n-1)x = (2(n-1)-4)90^0$$
$$x = \frac{n-3}{n-1}180^0$$

For (n+1) sided regular polygon:

Let measure of each angle is y.

Therefore

$$(n+2)y = (2(n+2)-4)90^0$$
$$y = \frac{n}{n+2}180^0$$

Now we have

$$y - x = 6^0$$
$$\frac{n}{n+2}180^0 - \frac{n-3}{n-1}180^0 = 6^0$$
$$\frac{n}{n+2} - \frac{n-3}{n-1} = \frac{1}{30}$$
$$30n(n-1) - 30(n-3)(n+2) = (n+2)(n-1)$$
$$-30n + 30n + 180 = n^2 + n - 2$$
$$n^2 + n - 182 = 0$$
$$(n-13)(n+14) = 0$$
$$n = 13, -14$$

Thus the value of n is 13.

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Solution 11:

(i)

Let the measure of each exterior angle is x and the number of sides is n.

Therefore we can write:

$$n = \frac{360^0}{x}$$

Now we have

$$x + x + 90^0 = 180^0$$
$$2x = 90^0$$
$$x = 45^0$$

(ii)

Thus the number of sides in the polygon is:

$$n = \frac{360^0}{45^0}$$
$$= 8$$

Exercise 14(B)

Solution 1:

(i)True.

This is true, because we know that a rectangle is a parallelogram. So, all the properties of a parallelogram are true for a rectangle. Since the diagonals of a parallelogram bisect each other, the same holds true for a rectangle.

(ii)False

This is not true for any random quadrilateral. Observe the quadrilateral shown below.

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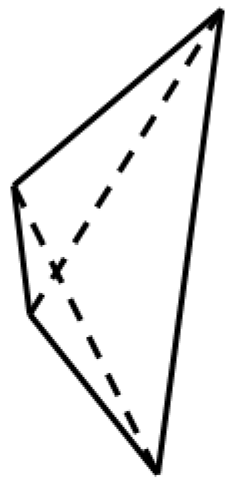
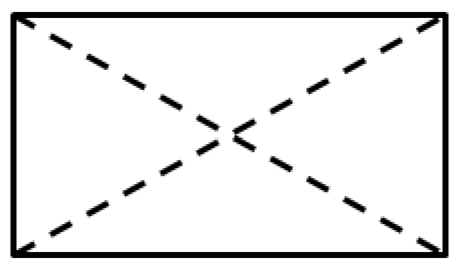


Clearly the diagonals of the given quadrilateral do not bisect each other. However, if the quadrilateral was a special quadrilateral like a parallelogram, this would hold true.

(iii)False

Consider a rectangle as shown below.



It is a parallelogram. However, the diagonals of a rectangle do not intersect at right angles, even though they bisect each other.

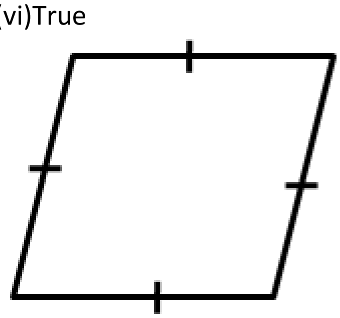
(iv)True

Since a rhombus is a parallelogram, and we know that the diagonals of a parallelogram bisect each other, hence the diagonals of a rhombus too, bisect other.

(v)False

This need not be true, since if the angles of the quadrilateral are not right angles, the quadrilateral would be a rhombus rather than a square.

(vi)True



A parallelogram is a quadrilateral with opposite sides parallel and equal.

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Since opposite sides of a rhombus are parallel, and all the sides of the rhombus are equal, a rhombus is a parallelogram.

(vii)False

This is false, since a parallelogram in general does not have all its sides equal. Only opposite sides of a parallelogram are equal. However, a rhombus has all its sides equal. So, every parallelogram cannot be a rhombus, except those parallelograms that have all equal sides.

(viii)False

This is a property of a rhombus. The diagonals of a rhombus need not be equal.

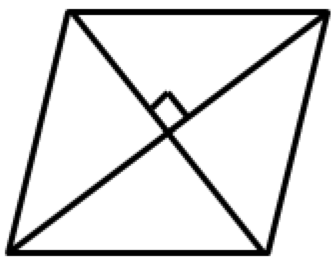
(ix)True

A parallelogram is a quadrilateral with opposite sides parallel and equal.

A rhombus is a quadrilateral with opposite sides parallel, and all sides equal.

If in a parallelogram the adjacent sides are equal, it means all the sides of the parallelogram are equal, thus forming a rhombus.

(x)False



Observe the above figure. The diagonals of the quadrilateral shown above bisect each other at right angles, however the quadrilateral need not be a square, since the angles of the quadrilateral are clearly not right angles.

Solution 2:

From the given figure we conclude that

$\angle A + \angle D = 180°$ [since consecutive angles are supplementary]

$\frac{\angle A}{2} + \frac{\angle D}{2} = 90°$

Again from the ΔADM

$\frac{\angle A}{2} + \frac{\angle D}{2} + \angle M = 180°$

$\Rightarrow 90° + \angle M = 180°$ $\left[\sin ce \frac{\angle A}{2} + \frac{\angle D}{2} = 90°\right]$

$\Rightarrow \angle M = 90°$

Hence $\angle AMD = 90°$

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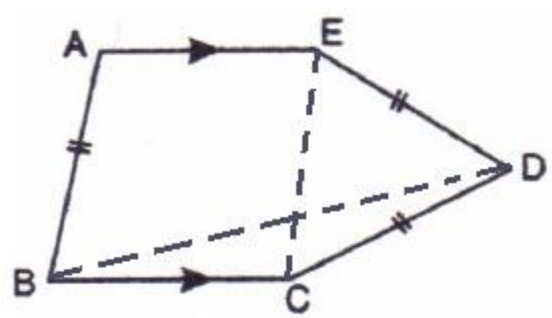
Solution 3:

In the given figure



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Given that AE = BC

We have to find $\angle AEC$ $\angle BCD$

Let us join EC and BD.

In the quadrilateral AECB

AE = BC and AB = EC

also AE || BC

$\Rightarrow$ AB || EC

So quadrilateral is a parallelogram.

In parallelogram consecutive angles are supplementary

$\Rightarrow \angle A + \angle B = 180^\circ$

$\Rightarrow 102^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = 78^\circ$

In parallelogram opposite angles are equal

$\Rightarrow \angle A = \angle BEC$ and $\angle B = \angle AEC$

$\Rightarrow \angle BEC = 102^\circ$ and $\angle AEC = 78^\circ$

Now consider ΔECD

EC = ED = CD [Since AB = EC]

Therefore ΔECD is an equilateral triangle.

$\Rightarrow \angle ECD = 60^\circ$

$\angle BCD = \angle BEC + \angle ECD$

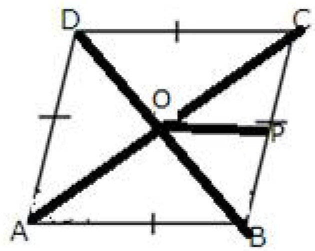
$\Rightarrow \angle BCD = 102^\circ + 60^\circ$

$\Rightarrow \angle BCD = 162^\circ$

Therefore $\angle AEC = 78^\circ$ and $\angle BCD = 162^\circ$

Solution 4:

Given ABCD is a square and diagonals meet at O.P is a point on BC such that OB=BP



In the

ΔBOC and ΔDOC

$\Rightarrow BD=BD$ [common side]

$\Rightarrow BO=CO$

ÞOD=OC [since diagonals cuts at O]

$\Delta BOC \cong \Delta DOC$ [by SSS]

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Therefore

$\angle BOC = 90°$

NOW

$\angle POC = 22.5$

$\angle BOP = 67.5$ $\left[\text{since } \angle \text{BOC}{=}67.5°{+}22.5°\ \right]$

Again

ΔBDC

$\angle BDC = 45°$ $\left[\text{since } \angle B = 45°, \angle C = 90°\right]$

Therefore

$\angle BDC = 2\angle POC$

AGAIN

$\angle BOP = 67.5°$

$\Rightarrow \angle BOP = 2\angle POC$

Hence proved that

i) $\angle$PC = $\left(22\frac{1}{2}^{\circ}\right)$

(ii) $\angle$BDC = 2 $\angle$POC

(iii) $\angle$BOP = 3 $\angle$CPO

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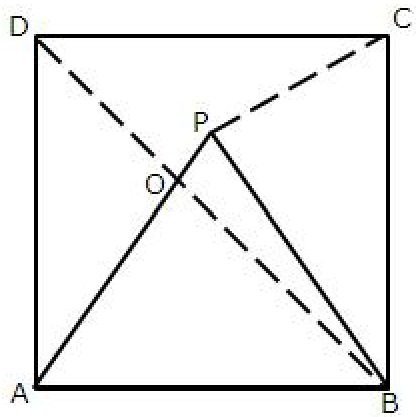
Solution 5:



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In the given figure ΔAPB is an equilateral triangle

Therefore all its angles are $60°$

Again in the

ΔADB

$\angle ABD = 45°$

$\angle AOB = 180° - 60° - 45°$

$= 75°$

Again

ΔBPC

$\Rightarrow \angle BPC = 75°$ [Since BP = CB]

Now

$\angle C = \angle BCP + \angle PCD$

$\Rightarrow \angle PCD = 90° - 75°$

$\Rightarrow \angle PCD = 15°$

Therefore

$\angle APC = 60° + 75°$

$\Rightarrow \angle APC = 135°$

$\Rightarrow Reflex\ \angle APD = 360° - 135° = 225°$

(i) $\angle AOB = 75°$

(ii) $\angle BPC = 75°$

(iii) $\angle PCD = 15°$

(iv) Reflex $\angle APD = 225°$

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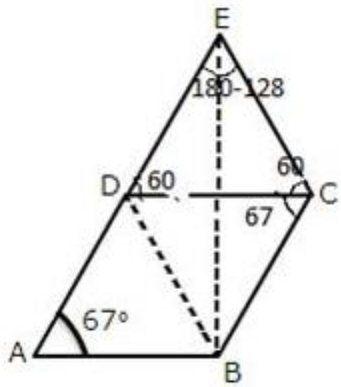
Solution 6:

Given that the figure ABCD is a rhombus with angle A = 67°



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In the rhombus We have

$\angle A = 67^\circ = \angle C$ [Opposite angles]

$\angle A + \angle D = 180^\circ$ [Consecutive angles are supplementary]

$\Rightarrow \angle D = 113^\circ$

$\Rightarrow \angle ABC = 113^\circ$

Consider $\triangle DBC$,

$DC = CB$ [Sides of rhombous]

So $\triangle DBC$ is an isoscales triangle

$\Rightarrow \angle CDB = \angle CBD$

Also,

$\angle CDB + \angle CDB + \angle BCD = 180^\circ$

$\Rightarrow 2\angle CBD = 113^\circ$

$\Rightarrow \angle CDB = \angle CBD = 56.5^\circ$............$(i)$

Consider $\triangle DCE$,

$EC = CB$

So $\triangle DCE$ is an isoscales triangle

$\Rightarrow \angle CBE = \angle CEB$

Also,

$\angle CBE + \angle CEB + \angle BCE = 180^\circ$

$\Rightarrow 2\angle CBE = 53^\circ$

$\Rightarrow \angle CDE = 26.5^\circ$

From (i)

$\angle CBD = 56.5^\circ$

$\Rightarrow \angle CBE + \angle DBE = 56.5^\circ$

$\Rightarrow 26.5^\circ + \angle DBE = 56.5^\circ$

$\Rightarrow \angle DBE = 30.5^\circ$

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Solution 7:

(i)ABCD is a parallelogram

Therefore

AD=BC
AB=DC

Thus

$4y = 3x - 3$ [since AD=BC]
$\Rightarrow 3x - 4y = 3$ (i)
$6y + 2 = 4x$ [since AB=DC]
$4x - 6y = 2$ (ii)

Solving equations (i) and (ii) we have

x=5

y=3

(ii)

In the figure ABCD is a parallelogram

$\angle A = \angle C$
$\angle B = \angle D$ [since opposite angles are equal]

Therefore

$7y = 6y + 3y - 8^{\circ}$ (i) [$Since\ \angle A = \angle C$]
$4x + 20^{\circ} = 0$ (ii)

Solving (i), (ii) we have

$X = 12^{\circ}$
$Y = 16^{\circ}$

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Solution 8:

Given that the angles of a quadrilateral are in the ratio $3:4:5:6$ Let the angles be $3x, 4x, 5x, 6x$

$$3x+4x+5x+6x=360°$$
$$\Rightarrow x=\frac{360°}{18}$$
$$\Rightarrow x=20°$$

Therefore the angles are

$$3\times 20=60°,$$
$$4\times 20=80°,$$
$$5\times 20=100°,$$
$$6\times 20=120°$$

Since all the angles are of different degrees thus forms a trapezium

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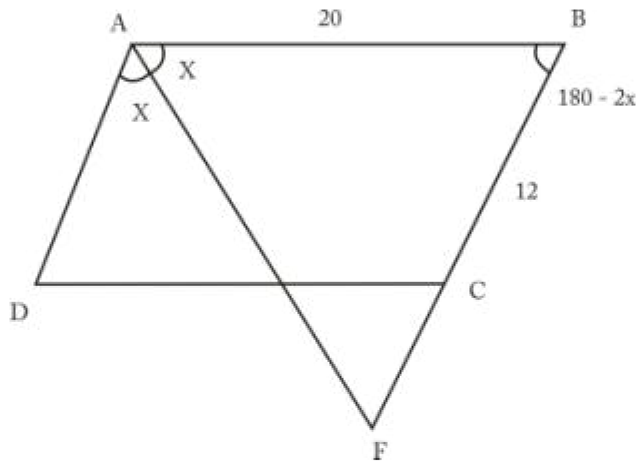
Solution 9:



Given AB = 20 cm and AD = 12 cm.

From the above figure, it's evident that ABF is an isosceles triangle with angle BAF = angle BFA = x

So AB = BF = 20

BF = 20

BC + CF = 20

CF = 20 - 12 = 8 cm

Solution 10:

We know that AQCP is a quadrilateral. So sum of all angles must be 360.

∴ x + y + 90 + 90 = 360

x + y = 180

Given x:y = 2:1

So substitute x = 2y

3y = 180

y = 60

x = 120

We know that angle C = angle A = x = 120

Angle D = Angle B = 180 – x = 180 – 120 = 60

Hence, angles of parallelogram are 120, 60, 120 and 60.

Exercise 14(C)

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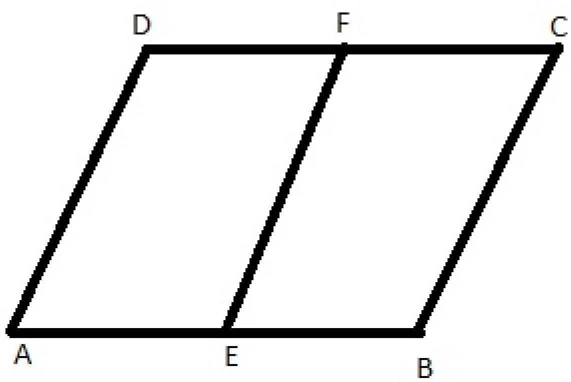
Solution 1:

Let us draw a parallelogram $ABCD$ Where F is the midpoint

Of side DC of parallelogram $ABCD$

To prove: $AEFD$ is a parallelogram



Proof:

Therefore $ABCD$

$$AB \parallel DC$$
$$BC \parallel AD$$
$$AB = DC$$
$$\frac{1}{2}AB = \frac{1}{2}DC$$
$$AE = DF$$

Also AD|| EF

therefore AEFC is a parallelogram.

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Solution 2:

GIVEN: ABCD is a parallelogram where the diagonal BD bisects

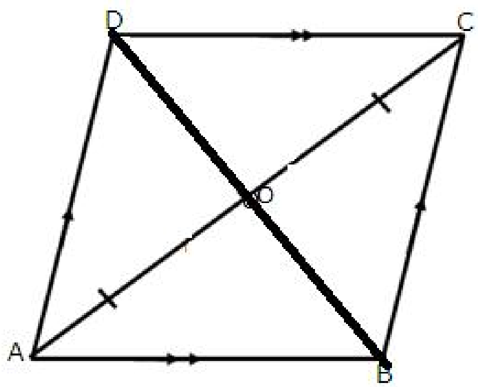
parallelogram ABCD at angle B and D

TO PROVE: ABCD is a rhombus

Proof : Let us draw a parallelogram ABCD where the diagonal BD bisects the parallelogram at angle B and D

Consruction :Let us join AC as a diagonal of the parallelogram

ABCD



Since $ABCD$ is a parallelogram

Therefore

$AB = DC$
$AD = BC$

Diagonal BD bisects angle B and D

So $\angle COD = \angle DOA$

Again AC also bisects at A and C

Therefore $\angle AOB = \angle BOC$

Thus ABCD is a rhombus.

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Solution 3:

Given ABCD is a parallelogram and AE=EF=FC.

We have to prove at first that DEBF is a parallelogram.

Proof :From ΔADE and ΔBCF

AE=FC
AD=BC
$\angle D = \angle B$

$\Delta ADE \cong \Delta BCF$ [SAS]

Therefore DE=FB

DC=EF [since AE+EF+FC=AC and AE=EF=FC]

Therefore DEBF is a parallelogram.

So DE || FB

Hence proved

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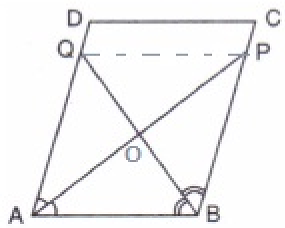
Solution 4:



Let us join PQ.

Consider the $\triangle AOQ$ and $\triangle BOP$
$\angle AOQ = \angle BOP$ [opposite angles]
$\angle OAQ = \angle BPO$ [alternate angles]
$\Rightarrow \triangle AOQ \cong \triangle BOP$ [AA test]

$Hence\ AQ = BP$

Consider the $\triangle QOP$ and $\triangle AOB$
$\angle AOB = \angle QOP$ [opposite angles]
$\angle OAB = \angle APQ$ [alternate angles]
$\Rightarrow \triangle QOP \cong \triangle AOB$ [AA test]

$Hence\ PQ = AB = CD$

Consider the quadrilateral $QPCD$
$DQ = CP\ and\ DQ \parallel CP$ [Since $AD = BC\ and\ AD \parallel BC$]
Also $QP = DC\ and\ AB \parallel QP \parallel DC$

$Hence\ quadrilateral\ QPCD$ is a parallelogram.

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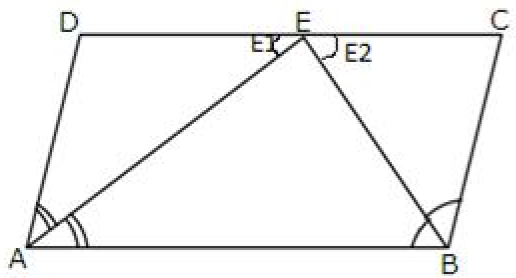
Solution 5:

Given ABCD is a parallelogram

To prove: AB = 2BC



Proof: ABCD is a parallelogram

$\angle A + \angle D = \angle B + \angle C = 180°$

From the ΔAEB we have

$\Rightarrow \dfrac{\angle A}{2} + \dfrac{\angle B}{2} + \angle E = 180°$

$\Rightarrow \angle A - \dfrac{\angle A}{2} + \angle D + \angle E1 = 180°$ [*taking* E1 as new angle]

$\Rightarrow \angle A + \angle D + \angle E1 = 180° + \dfrac{\angle A}{2}$

$\Rightarrow \angle E1 = \dfrac{\angle A}{2}$ [*Since* $\angle A + \angle D = 180°$]

Again,

similarly,

$\angle E2 = \dfrac{\angle B}{2}$

NOW

AB=DE+EC

=AD+BC

=2BC [since AD=BC]

Hence proved

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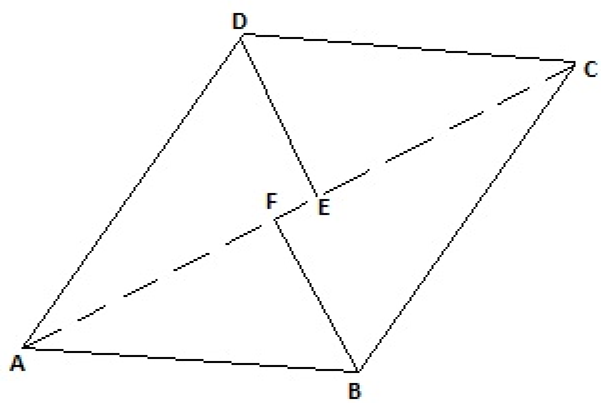
Solution 6:

Given ABCD is a parallelogram. The bisectors of $\angle ADC$ and $\angle BCD$ meet at E. The bisectors of $\angle ABC$ and $\angle BCD$ meet at F

From the parallelogram ABCD we have



$\angle ADC + \angle BCD = 180°$ [sum of adjacent angles of a parallelogram]

$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90°$

$\Rightarrow \angle EDC + \angle ECD = 90°$

In triangle ECD sum of angles $= 180°$

$\Rightarrow \angle EDC + \angle ECD + \angle CED = 180°$

$\Rightarrow \angle CED = 90°$

Similarly taking triangle BCF it can be prove that $\angle BFC = 90°$

Now since

$\angle BFC = \angle CED = 90°$

Therefore the lines DE and BF are parallel

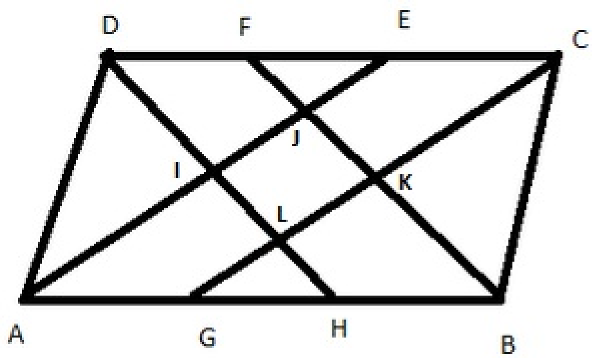
Hence proved

Solution 7:

Given: $ABCD$ is a parallelogram

AE bisects $\angle BAD$
BF bisects $\angle ABC$
CG bisects $\angle BCD$
DH bisecsts $\angle ADC$

TO PROVE: $LKJI$ is a rectangle



Proof :

$\angle BAD + \angle ABC = 180°$ [adjacent angles of a parallelogram are supplementary]

$\angle BAJ = \frac{1}{2}\angle BAD$ [AE bisects ĐBAD]

$\angle ABJ = \frac{1}{2}\angle ABC$ [DH bisect ĐABC]

$\angle BAJ + \angle ABJ = 90°$ [halves of supplementary angles are complementary]

ΔABJ is a right triangle because its acute interior angles are complementary.

Similarly

$\angle DLC = 90°$
$\angle AID = 90°$
Then $\angle JIL = 90°$ because $\angle AID$ and $\angle JIL$ are vertical angles

since 3 angles of quadrilateral $LKJI$ are right angles,si is the 4th one and so $LKJI$ is a rectangle ,since its interior angles are all right angles

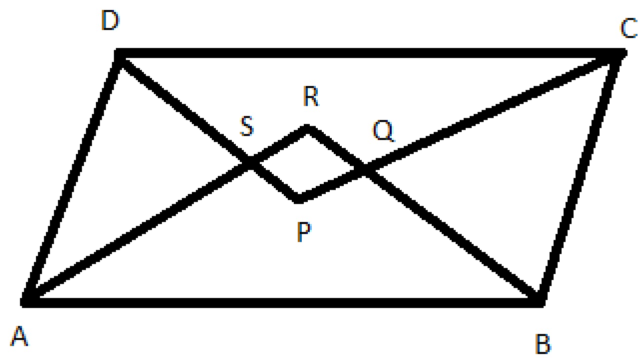
Hence proved

Solution 8:

Given: A parallelogram $ABCD$ in which AR, BR, CP, DP

Are the bisects of $\angle A, \angle B, \angle C, \angle D$ respectively forming quadrilaterals $PQRS$.

To prove: $PQRS$ is a rectangle



Proof :

$\angle DCB + \angle ABC = 180°$ [co-interior angles of parallelogram are supplementary]

$\Rightarrow \frac{1}{2}\angle DCB + \frac{1}{2}\angle ABC = 90°$

$\Rightarrow \angle 1 + \angle 2 = 90°$

Also in

$\Delta CQB, \angle 1 + \angle 2 + \angle CQB = 180°$

From the above equation we get

$\angle CQB = 180° - 90° = 90°$

$\angle RQP = 90°$ [$\angle CQB = \angle RQP$, *vertically* opposite angles]

$\angle QRP = \angle RSP = \angle SPQ = 90°$

Hence PQRS is a rectangle

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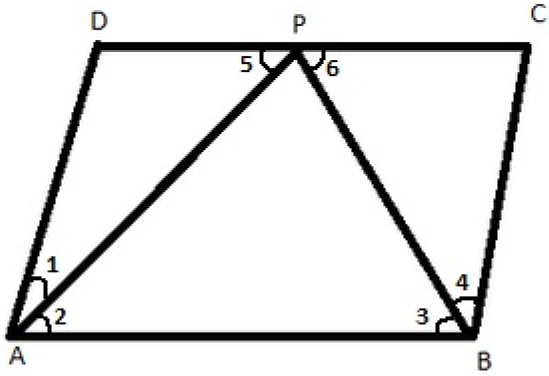
Solution 9:



(i)Let $AD = x$

$AB = 2AD = 2x$

Also AP is the bisector $\angle A$

$\angle 1 = \angle 2$

Now ,

$\angle 2 = \angle 5$ [alternate angles]

Therefore $\angle 1 = \angle 5$

Now

$AP = DP = x$ [sides opposite to equal angles are also equal]

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Therefore

$AB=CD$ [opposite sides of parallelogram are equal]
$CD = 2x$
$\Rightarrow DP+PC=2x$
$\Rightarrow x+PC=2x$
$\Rightarrow PC = x$

Also, $BC=x$

In ΔBPC
$\Rightarrow \angle 6=\angle 4$ [*angles* opposite to equal sides are equal]
$\Rightarrow \angle 6 = \angle 3$

Therefore $\angle 3 = \angle 4$

Hence BP bisect $\angle B$

(ii)

Opposite angles are supplementary

Therefore

$\angle 1+\angle 2+\angle 3+\angle 4=180°$
$\Rightarrow 2\angle 2+2\angle 3=180° \begin{bmatrix}\angle 1=\angle 2 \\ \angle 3=\angle 4\end{bmatrix}$
$\Rightarrow \angle 2+\angle 3=90°$

ΔAPB
$\angle 2+\angle 3\angle APB=180°$
$\Rightarrow \angle APB=180°-90°$ [by angle sum property]
$\Rightarrow \angle APB=90°$

Hence proved

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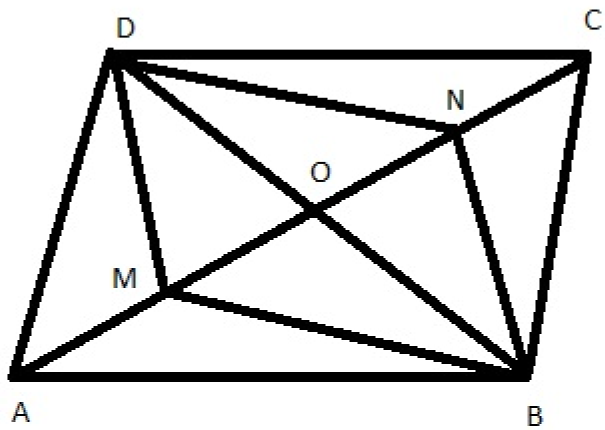
Solution 10:

Points M and N are taken on the diagonal AC of a parallelogram $ABCD$ such that $AM=CN$.

Prove that $BMDN$ is a parallelogram



CONSTRUCTION: Join B to D to meet AC in O.

PROOF: We know that the diagonals of parallelogram bisect each other.

Now, AC and BD bisect each other at O.

$OC=OA$

$AM=CN$

$\Rightarrow OA-AM=OC-CN$

$\Rightarrow OM=ON$

Thus in a quadrilateral $BMDN$, diagonal BD and MN are such that $OM=ON$ and $OD=OB$

Therefore the diagonals AC and PQ bisect each other.

Hence $BMDN$ is a parallelogram

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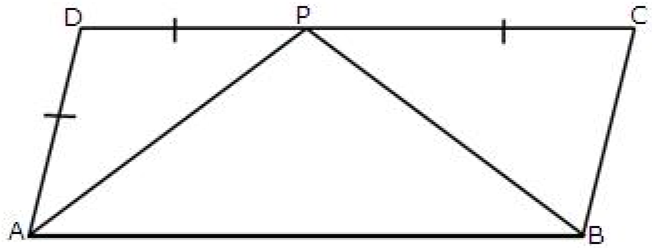
Solution 11:



Consider ΔADP and ΔBCP

$AD=BC$ [since ABCD is a parallelogram]
$DC=AB$ [since ABCD is a parallelogram]
$\angle A = \angle C$ [opposite angles]
$\Delta ADP \cong \Delta BCP$ [SAS]

Therefore $AP=BP$

AP bisects $\angle A$
BP bisects $\angle B$

In ΔAPB
$AP=PB$
$\angle APB = \angle DAP + \angle BCP$

Hence proved

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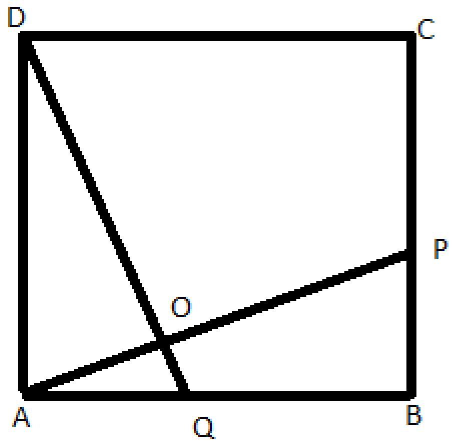
Solution 12:



ABCD is a square and $AP=PQ$

Consider $\triangle DAQ$ and $\triangle ABP$

$\angle DAQ = \angle ABP = 90^{\circ}$

$DQ = AP$

$AD = AB$

$\triangle DAQ \cong \triangle ABP$

$\Rightarrow \angle PAB = \angle QDA$

Now,

$\angle PAB + \angle APB = 90^{\circ}$

also $\angle QDA + \angle APB = 90^{\circ}$ $[\angle PAB = \angle QDA]$

Consider $\triangle AOQ$ By ASP

$\angle QDA + \angle APB + \angle AOD = 180^{\circ}$

$\Rightarrow 90^{\circ} + \angle AOD = 180^{\circ}$

$\Rightarrow \angle AOD = 90^{\circ}$

Hence AP and DQ are perpendicular.

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Solution 13:

Given: ABCD is quadrilateral,

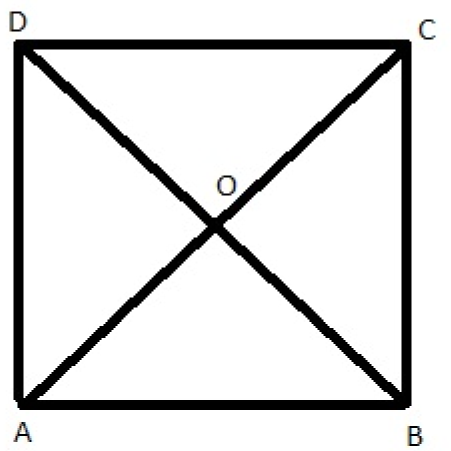
AB=AD

CB=CD

To prove: (i) AC bisects angle BAD.

(ii) AC is perpendicular bisector of BD.

Proof :



In ΔABC and ΔADC

AB=AD [given]

CB=CD [given]

AC=AC [common side]

$\Delta ABC \cong \Delta ADC$ [SSS]

Therefore AC bisects $\angle BAD$

OD=OB

OA=OA [diagonals bisect each other at O]

Thus AC is perpendicular bisector of BD

Hence proved

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Solution 14:

Given ABCD is a trapezium, AB || DC and AD=BC

To prove(i) ∠DAB = ∠CBA

(ii) ∠ADC = ∠BCD

(iii) AC = BD

(iv) OA = OB and OC = OD

Proof :(i) Since AD || CE and transversal AE cuts them at A and E respectively.

Therefore, $\angle A + \angle B = 180°$

Since AB || CD and AD || BC

Therefore ABCD is a parallelogram

$\angle A = \angle C$

$\angle B = \angle D$ [since ABCD is a parallelogram]

Therefore $\angle DAB = \angle CBA$

$\angle ADC = \angle BCD$

In ΔABC and ΔBAD ,we have

BC=AD [given]

AB=BA [common]

$\angle A = \angle B$ [proved]

$\Delta ABC \cong \Delta BAD$ [SAS]

Since ΔABC ≅ ΔBAD
Therefore AC=BD [corresponding parts of congruent triangles are equal]

Again OA=OB
OC=OD [since diagonals bisect each other at O]

Hence proved

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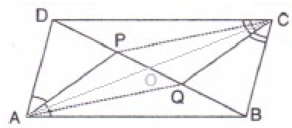
Solution 15:



Join AC to meet BD in O

We know that the diagonals of a parallelogram bisect each other. Therefore AC and BD bisect each other at O.

Therefore

$OB=OD$
But
$BQ=DP$

$OB-PQ=OD-DP$
$\Rightarrow OQ=OP$

Thus in a quadrilateral $APCQ$ diagonals AC and PQ such that $OQ=OP$ and $OA=OC$. Since diagonals AC and PQ bisect each other.

Hence $APCQ$ is a parallelogram

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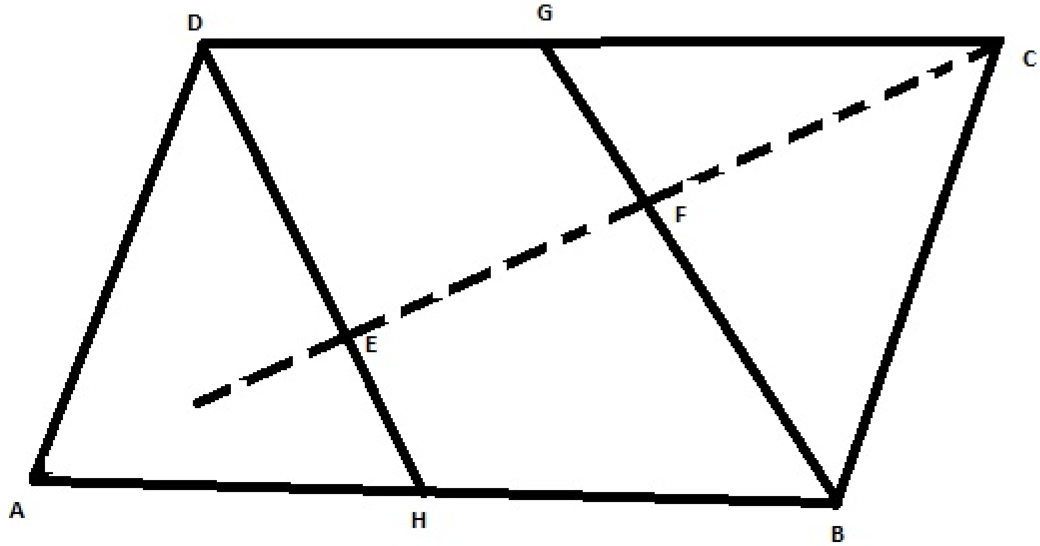
Solution 16:



ABCD is a parallelogram ,the bisectors of $\angle ADC$ and $\angle BCD$ meet at a point E and the bisectors of $\angle BCD$ AND $\angle ABC$ meet at F.

We have to prove that the $\angle CED = 90°$ and $\angle CFG = 90°$

Proof :In the parallelogram ABCD

$\angle ADC + \angle BCD = 180°$ [sum of adjacent angles of a parallelogram]

$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90°$

$\Rightarrow \angle EDC + \angle ECD + \angle CED = 180°$

$\Rightarrow \angle CED = 90°$

Similarly taking triangle BCF it can be proved that $\angle BFC = 90°$

Also $\angle BFC + \angle CFG = 180°$ [adjacent angles on a line]

$\Rightarrow \angle CFG = 90°$

Now since $\angle CFG = \angle CED = 90°$ [it means that the lines DE and BG are parallel]

Hence proved

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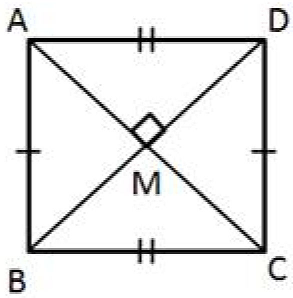
Solution 17:



To prove : ABCD is a square,
that is, to prove that sides of the quadrilateral are equal and each angle of the quadrilateral is 90°.
ABCD is a rectangle,
$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90°$ and diagonals bisect each other
that is, MD = BM...(i)
Consider ΔAMD and ΔAMB,
MD = BM (from (i))
$\angle AMD = \angle AMB = 90°$ (given)
AM = AM (common side)
$\Delta AMD \cong \Delta AMB$ (SAS congruence criterion)
$\Rightarrow AD = AB$ (cpctc)
Since ABCD is a rectangle, AD = BC and AB = CD
Thus, $AB = BC = CD = AD$ and $\angle A = \angle B = \angle C = \angle D = 90°$
$\Rightarrow$ ABCD is a square.

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Solution 18:

ABCD is a parallelogram

$\Rightarrow$ opposite angles of a parallelogram are congruent

$\Rightarrow \angle DAB = \angle BCD$ and $\angle ABC = \angle ADC = 120°$

In ABCD,

$\angle DAB + \angle BCD + \angle ABC + \angle ADC = 360°$

.......(sum of the measures of angles of a quadrilateral)

$\Rightarrow \angle BCD + \angle BCD + 120° + 120° = 360°$

$\Rightarrow 2\angle BCD = 360° - 240°$

$\Rightarrow 2\angle BCD = 120°$

$\Rightarrow \angle BCD = 60°$

PQRS is a parallelogram

$\Rightarrow \angle PQR = \angle PSR = 70°$

In ΔCMS,

$\angle CMS + \angle CSM + \angle MCS = 180°$ ....(angle sum property)

$\Rightarrow x + 70° + 60° = 180°$

$\Rightarrow x = 50°$

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Solution 19:

ABCD is a rhombus $\Rightarrow AD = CD$ and $\angle ADC = \angle ABC = 56°$

DCFE is a square $\Rightarrow ED = CD$ and $\angle FED = \angle EDC = \angle DCF = \angle CFE = 90°$

$\Rightarrow AD = CD = ED$

In ΔADE,

$AD = ED \Rightarrow \angle DAE = \angle AED \ldots(i)$

$\angle DAE + \angle AED + \angle ADE = 180°$

$\Rightarrow 2\angle DAE + 146° = 180° \quad \ldots(\text{Since } \angle ADE = \angle EDC + \angle ADC = 90° + 56° = 146°)$

$\Rightarrow 2\angle DAE = 34°$

$\Rightarrow \angle DAE = 17°$

$\Rightarrow \angle DEA = 17° \ldots(ii)$

In ABCD,

$\angle ABC + \angle BCD + \angle ADC + \angle DAB = 360°$

$\Rightarrow 56° + 56° + 2\angle DAB = 360°$ ($\because$ opposite angles of a rhombus are equal)

$\Rightarrow 2\angle DAB = 248°$

$\Rightarrow \angle DAB = 124°$

We know that diagonals of a rhombus, bisect its angles.

$\Rightarrow \angle DAC = \dfrac{124°}{2} = 62°$

$\Rightarrow \angle EAC = \angle DAC - \angle DAE = 62° - 17° = 45°$

Now, $\angle FEA = \angle FED - \angle DEA$

$= 90° - 17° \quad \ldots(\text{from (ii) and each angle of a square is } 90°)$

$= 73°$

We know that diagonals of a square bisect its angles.

$\Rightarrow \angle CED = \dfrac{90°}{2} = 45°$

So, $\angle AEC = \angle CED - \angle DEA$

$= 45° - 17°$

$= 28°$

Hence, $\angle DAE = 17°$, $\angle FEA = 73°$, $\angle EAC = 45°$ and $\angle AEC = 28°$.