



## Selina ICSE Solutions for Class 9 Maths Chapter 8 Logarithms

### Exercise 8(A)

#### Solution 1:

(i)

$$5^3 = 125$$

$$\Rightarrow \log_5 125 = 3 \quad [a^b = c \Rightarrow \log_a c = b]$$

(ii)

$$3^{-2} = \frac{1}{9}$$

$$\Rightarrow \log_3 \frac{1}{9} = -2 \quad [a^b = c \Rightarrow \log_a c = b]$$

(iii)

$$10^{-3} = 0.001$$

$$\Rightarrow \log_{10} 0.001 = -3 \quad [a^b = c \Rightarrow \log_a c = b]$$

(iv)

$$(81)^{\frac{3}{4}} = 27$$

$$\Rightarrow \log_{81} 27 = \frac{3}{4} \quad [\text{By definition of logarithm, } a^b = c \Rightarrow \log_a c = b]$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 2:

(i)

$$\log_8 0.125 = -1$$

$$\Rightarrow 8^{-1} = 0.125 \quad [\log_a c = b \Rightarrow a^b = c]$$

(ii)

$$\log_{10} 0.01 = -2$$

$$\Rightarrow 10^{-2} = 0.01 \quad [\log_a c = b \Rightarrow a^b = c]$$

(iii)

$$\log_a A = x$$

$$\Rightarrow a^x = A \quad [\log_a c = b \Rightarrow a^b = c]$$

(iv)

$$\log_{10} 1 = 0$$

$$\Rightarrow 10^0 = 1 \quad [\log_a c = b \Rightarrow a^b = c]$$

### Solution 3:

$$\log_{10} x = -2$$

$$\Rightarrow 10^{-2} = x \quad [\log_a c = b \Rightarrow a^b = c]$$

$$\Rightarrow x = 10^{-2}$$

$$\Rightarrow x = \frac{1}{10^2}$$

$$\Rightarrow x = \frac{1}{100}$$

$$\Rightarrow x = 0.01$$



## Solution 4:

(i)

$$\text{Let } \log_{10} 100 = x$$

$$\therefore 10^x = 100$$

$$\Rightarrow 10^x = 10 \times 10$$

$$\Rightarrow 10^x = 10^2$$

$$\Rightarrow x = 2 \quad [\text{if } a^m = a^n; \text{ then } m=n]$$

$$\therefore \log_{10} 100 = 2$$

(ii)

$$\text{Let } \log_{10} 0.1 = x$$

$$\therefore 10^x = 0.1$$

$$\Rightarrow 10^x = \frac{1}{10}$$

$$\Rightarrow 10^x = 10^{-1}$$

$$\Rightarrow x = -1 \quad [\text{if } a^m = a^n; \text{ then } m=n]$$

$$\therefore \log_{10} 0.1 = -1$$

(iii)

$$\text{Let } \log_{10} 0.001 = x$$

$$\therefore 10^x = 0.001$$

$$\Rightarrow 10^x = \frac{1}{1000}$$

$$\Rightarrow 10^x = \frac{1}{10^3}$$

$$\Rightarrow 10^x = 10^{-3}$$

$$\Rightarrow x = -3 \quad [\text{if } a^m = a^n; \text{ then } m=n]$$

$$\therefore \log_{10} 0.001 = -3$$

(iv)

$$\text{Let } \log_4 32 = x$$

$$\therefore 4^x = 32$$

$$\Rightarrow (2^2)^x = 2 \times 2 \times 2 \times 2 \times 2$$

$$\Rightarrow 2^{2x} = 2^5$$

$$\Rightarrow 2x = 5 \quad [\text{if } a^m = a^n; \text{ then } m=n]$$

$$\Rightarrow x = \frac{5}{2}$$

$$\therefore \log_4 32 = \frac{5}{2}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(v)

$$\text{Let } \log_2 0.125 = x$$

$$\therefore 2^x = 0.125$$

$$\Rightarrow 2^x = \frac{125}{1000}$$

$$\Rightarrow 2^x = \frac{1}{8}$$

$$\Rightarrow 2^x = 8^{-1}$$

$$\Rightarrow 2^x = (2 \times 2 \times 2)^{-1}$$

$$\Rightarrow 2^x = (2^3)^{-1}$$

$$\Rightarrow 2^x = 2^{-3}$$

$$\Rightarrow x = -3 \text{ [if } a^m = a^n \text{; then } m=n]$$

$$\therefore \log_2 0.125 = -3$$

(vi)

$$\text{Let } \log_4 \frac{1}{16} = x$$

$$\therefore 4^x = \frac{1}{16}$$

$$\Rightarrow 4^x = \frac{1}{4 \times 4}$$

$$\Rightarrow 4^x = (4 \times 4)^{-1}$$

$$\Rightarrow 4^x = (4^2)^{-1}$$

$$\Rightarrow 4^x = 4^{-2}$$

$$\Rightarrow x = -2 \text{ [if } a^m = a^n \text{; then } m=n]$$

$$\therefore \log_4 \frac{1}{16} = -2$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(vii)

$$\text{Let } \log_9 27 = x$$

$$\therefore 9^x = 27$$

$$\Rightarrow (3 \times 3)^x = 3 \times 3 \times 3$$

$$\Rightarrow (3^2)^x = (3^3)$$

$$\Rightarrow 3^{2x} = (3^3)$$

$$\Rightarrow 2x = 3 \text{ [if } a^m = a^n; \text{ then } m=n]$$

$$\Rightarrow x = \frac{3}{2}$$

$$\therefore \log_9 27 = \frac{3}{2}$$

(viii)

$$\text{Let } \log_{27} \frac{1}{81} = x$$

$$\therefore 27^x = \frac{1}{81}$$

$$\Rightarrow (3 \times 3 \times 3)^x = \frac{1}{3 \times 3 \times 3 \times 3}$$

$$\Rightarrow (3^3)^x = \frac{1}{3^4}$$

$$\Rightarrow (3^3)^x = (3^4)^{-1}$$

$$\Rightarrow 3^{3x} = (3^{-4})$$

$$\Rightarrow 3x = -4 \text{ [if } a^m = a^n; \text{ then } m=n]$$

$$\Rightarrow x = \frac{-4}{3}$$

$$\therefore \log_{27} \frac{1}{81} = \frac{-4}{3}$$



### Solution 5:

(i)

Consider the equation

$$\log_{10} x = a$$

$$\Rightarrow 10^a = x$$

Thus the statement,  $10^a = a$  is false

(ii)

Consider the equation

$$x^y = z$$

$$\Rightarrow \log_x z = y$$

Thus the statement,  $\log_z x = y$  is false

(iii)

Consider the equation

$$\log_2 8 = 3$$

$$\Rightarrow 2^3 = 8 \dots (1)$$

Now consider the equation

$$\log_8 2 = \frac{1}{3}$$

$$\Rightarrow 8^{\frac{1}{3}} = 2$$

$$\Rightarrow (2^3)^{\frac{1}{3}} = 2 \dots (2)$$

Both the equations (1) and (2) are correct

Thus the given statements,  $\log_2 8 = 3$  and  $\log_8 2 = \frac{1}{3}$  are true



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 6:

(i)

Consider the equation

$$\log_3 x = 0$$

$$\Rightarrow 3^0 = x$$

$$\Rightarrow 1 = x \text{ or } x=1$$

(ii)

Consider the equation

$$\log_x 2 = -1$$

$$\Rightarrow x^{-1} = 2$$

$$\Rightarrow \frac{1}{x} = 2$$

$$\Rightarrow x = \frac{1}{2}$$

(iii)

Consider the equation

$$\log_9 243 = x$$

$$\Rightarrow 9^x = 243$$

$$\Rightarrow (3^2)^x = 3^5$$

$$\Rightarrow 3^{2x} = 3^5$$

$$\Rightarrow 2x=5$$

$$\Rightarrow x = \frac{5}{2}$$

$$\Rightarrow x = 2\frac{1}{2}$$



(iv)

Consider the equation

$$\log_5 (x - 7) = 1$$

$$\Rightarrow 5^1 = x - 7$$

$$\Rightarrow 5 = x - 7$$

$$\Rightarrow x = 5 + 7$$

$$\Rightarrow x = 12$$

(v)

Consider the equation

$$\log_4 32 = x - 4$$

$$\Rightarrow 4^{x-4} = 32$$

$$\Rightarrow (2^2)^{x-4} = 2^5$$

$$\Rightarrow 2^{2(x-4)} = 2^5$$

$$\Rightarrow 2x - 8 = 5$$

$$\Rightarrow 2x = 5 + 8$$

$$\Rightarrow 2x = 13$$

$$\Rightarrow x = \frac{13}{2}$$

$$\Rightarrow x = 6\frac{1}{2}$$

(vi)

Consider the equation

$$\log_7 (2x^2 - 1) = 2$$

$$\Rightarrow 7^2 = 2x^2 - 1$$

$$\Rightarrow 7 \times 7 = 2x^2 - 1$$

$$\Rightarrow 2x^2 - 1 - 49 = 0$$

$$\Rightarrow 2x^2 - 50 = 0$$

$$\Rightarrow 2x^2 = 50$$

$$\Rightarrow x^2 = \frac{50}{2}$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \pm\sqrt{25}$$

$$\Rightarrow x = 5 \text{ [neglecting the negative value]}$$





## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 7:

(i)

$$\text{Let } \log_{10} 0.01 = x$$

$$\Rightarrow 10^x = 0.01$$

$$\Rightarrow 10^x = \frac{1}{100}$$

$$\Rightarrow 10^x = \frac{1}{10 \times 10}$$

$$\Rightarrow 10^x = \frac{1}{10^2}$$

$$\Rightarrow 10^x = 10^{-2}$$

$$\Rightarrow x = -2$$

$$\text{Thus, } \log_{10} 0.01 = -2$$

(ii)

$$\text{Let } \log_2 \frac{1}{8} = x$$

$$\Rightarrow 2^x = \frac{1}{8}$$

$$\Rightarrow 2^x = \frac{1}{2 \times 2 \times 2}$$

$$\Rightarrow 2^x = \frac{1}{2^3}$$

$$\Rightarrow 2^x = 2^{-3}$$

$$\Rightarrow x = -3$$

$$\text{Thus, } \log_2 \frac{1}{8} = -3$$

(iii)

$$\text{Let } \log_5 1 = x$$

$$\Rightarrow 5^x = 1$$

$$\Rightarrow 5^x = 5^0$$

$$\Rightarrow x = 0$$

$$\text{Thus, } \log_5 1 = 0$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(iv)

$$\text{Let } \log_5 125 = x$$

$$\Rightarrow 5^x = 125$$

$$\Rightarrow 5^x = 5 \times 5 \times 5$$

$$\Rightarrow 5^x = 5^3$$

$$\Rightarrow x = 3$$

$$\text{Thus, } \log_5 125 = 3$$

(v)

$$\text{Let } \log_{16} 8 = x$$

$$\Rightarrow 16^x = 8$$

$$\Rightarrow (2 \times 2 \times 2 \times 2)^x = 2 \times 2 \times 2$$

$$\Rightarrow (2^4)^x = 2^3$$

$$\Rightarrow 2^{4x} = 2^3$$

$$\Rightarrow 4x = 3$$

$$\Rightarrow x = \frac{3}{4}$$

$$\text{Thus, } \log_{16} 8 = \frac{3}{4}$$

(vi)

$$\text{Let } \log_{0.5} 16 = x$$

$$\Rightarrow 0.5^x = 16$$

$$\Rightarrow \left(\frac{5}{10}\right)^x = 2 \times 2 \times 2 \times 2$$

$$\Rightarrow \left(\frac{1}{2}\right)^x = 2^4$$

$$\Rightarrow \frac{1}{2^x} = 2^4$$

$$\Rightarrow 2^{-x} = 2^4$$

$$\Rightarrow -x = 4$$

$$\Rightarrow x = -4$$

$$\text{Thus, } \log_{0.5} 16 = -4$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 8:

$$\log_a m = n$$

$$\Rightarrow a^n = m$$

$$\Rightarrow \frac{a^n}{a} = \frac{m}{a}$$

$$\Rightarrow a^{n-1} = \frac{m}{a}$$

### Solution 9:

$$\log_2 x = m \text{ and } \log_5 y = n$$

$$\Rightarrow 2^m = x \text{ and } 5^n = y$$

(i) Consider  $2^m = x$

$$\Rightarrow \frac{2^m}{2^3} = \frac{x}{2^3}$$

$$\Rightarrow 2^{m-3} = \frac{x}{8}$$

(ii) Consider  $5^n = y$

$$\Rightarrow (5^n)^3 = y^3$$

$$\Rightarrow 5^{3n} = y^3$$

$$\Rightarrow 5^{3n} \times 5^2 = y^3 \times 5^2$$

$$\Rightarrow 5^{3n+2} = 25y^3$$



### Solution 10:

Given that :

$$\log_2^x = a \text{ and } \log_3^y = a$$

$$\Rightarrow 2^a = x \text{ and } 3^a = y \quad \left[ \begin{array}{l} \text{Q } \log_a^m = n \\ \Rightarrow a^n = m \end{array} \right]$$

Now prime factorization of 72 is

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

Hence,

$$\begin{aligned} (72)^a &= (2 \times 2 \times 2 \times 3 \times 3)^a \\ &= (2^3 \times 3^2)^a \\ &= 2^{3a} \times 3^{2a} \\ &= (2^a)^3 \times (3^a)^2 \quad \left[ \begin{array}{l} \text{as } 2^a = x \\ 3^a = y \end{array} \right] \\ &= x^3 y^2 \end{aligned}$$

### Solution 11:

$$\log(x-1) + \log(x+1) = \log_2 1$$

$$\Rightarrow \log(x-1) + \log(x+1) = 0$$

$$\Rightarrow \log[(x-1)(x+1)] = 0$$

$$\Rightarrow (x-1)(x+1) = 1 \dots (\text{Since } \log 1 = 0)$$

$$\Rightarrow x^2 - 1 = 1$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow x = \pm\sqrt{2}$$

$-\sqrt{2}$  cannot be possible, since log of a negative number is not defined.

$$\text{So, } x = \sqrt{2}.$$

### Solution 12:

$$\log(x^2 - 21) = 2$$

$$\Rightarrow x^2 - 21 = 10^2$$

$$\Rightarrow x^2 - 21 = 100$$

$$\Rightarrow x^2 = 121$$

$$\Rightarrow x = \pm 11$$

### Exercise 8(B)



## Solution 1:

(i)

$$\begin{aligned}\log 36 &= \log(2 \times 2 \times 3 \times 3) \\ &= \log(2^2 \times 3^2) \\ &= \log(2^2) + \log(3^2) \quad [\log_e mn = \log_e m + \log_e n] \\ &= 2\log 2 + 2\log 3 \quad [\log_e m^n = n\log_e m]\end{aligned}$$

(ii)

$$\begin{aligned}\log 144 &= \log(2 \times 2 \times 2 \times 2 \times 3 \times 3) \\ &= \log(2^4 \times 3^2) \\ &= \log(2^4) + \log(3^2) \quad [\log_e mn = \log_e m + \log_e n] \\ &= 4\log 2 + 2\log 3 \quad [\log_e m^n = n\log_e m]\end{aligned}$$

(iii)

$$\begin{aligned}\log 4.5 &= \log \frac{45}{10} \\ &= \log \frac{5 \times 3 \times 3}{5 \times 2} \\ &= \log \frac{3^2}{2} \\ &= \log 3^2 - \log 2 \quad [\log_e \frac{m}{n} = \log_e m - \log_e n] \\ &= 2\log 3 - \log 2 \quad [\log_e m^n = n\log_e m]\end{aligned}$$

(iv)

$$\begin{aligned}\log \frac{26}{51} - \log \frac{91}{119} &= \log \frac{\frac{26}{51}}{\frac{91}{119}} \quad [\log_e m - \log_e n = \log_e \frac{m}{n}] \\ &= \log \frac{26}{51} \times \frac{119}{91} \\ &= \log \frac{2 \times 13}{3 \times 17} \times \frac{7 \times 17}{7 \times 13} \\ &= \log \frac{2}{3} \\ &= \log 2 - \log 3 \quad [\log_e \frac{m}{n} = \log_e m - \log_e n]\end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(v)

$$\begin{aligned} & \log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} \\ &= \log \frac{75}{16} - \log \left( \frac{5}{9} \right)^2 + \log \frac{32}{243} \quad [n \log_a m = \log_a m^n] \\ &= \log \frac{75}{16} - \log \frac{5}{9} \times \frac{5}{9} + \log \frac{32}{243} \\ &= \log \frac{75}{16} - \log \frac{25}{81} + \log \frac{32}{243} \\ &= \log \left( \frac{75}{16} \times \frac{81}{25} \right) + \log \frac{32}{243} \quad \left[ \log_a m - \log_a n = \log_a \frac{m}{n} \right] \\ &= \log \frac{75}{16} \times \frac{81}{25} + \log \frac{32}{243} \\ &= \log \frac{3 \times 25}{16} \times \frac{81}{25} + \log \frac{32}{243} \\ &= \log \frac{3 \times 81}{16} + \log \frac{32}{243} \\ &= \log \frac{243}{16} + \log \frac{32}{243} \\ &= \log \frac{243}{16} \times \frac{32}{243} \quad \left[ \log_a m + \log_a n = \log_a mn \right] \\ &= \log \frac{32}{16} \\ &= \log 2 \end{aligned}$$



## Solution 2:

(i)

Consider the given equation

$$2\log x - \log y = 1$$

$$\Rightarrow \log x^2 - \log y = 1$$

$$\Rightarrow \log \frac{x^2}{y} = \log 10$$

$$\Rightarrow \frac{x^2}{y} = 10$$

$$\Rightarrow x^2 = 10y$$

(ii)

Consider the given equation

$$2\log x + 3\log y = \log a$$

$$\Rightarrow \log x^2 + \log y^3 = \log a$$

$$\Rightarrow \log x^2 y^3 = \log a$$

$$\Rightarrow x^2 y^3 = a$$

(iii)

Consider the given equation

$$a\log x - b\log y = 2\log 3$$

$$\Rightarrow \log x^a - \log y^b = \log 3^2$$

$$\Rightarrow \log \frac{x^a}{y^b} = \log 9$$

$$\Rightarrow \frac{x^a}{y^b} = 9$$

$$\Rightarrow x^a = 9y^b$$



### Solution 3:

(i) Consider the given expression

$$\begin{aligned}\log 5 + \log 8 - 2\log 2 &= \log 5 + \log 8 \times 8 - \log 2^2 && [n \log_a m = \log_a m^n] \\ &= \log 5 \times 8 - \log 2^2 && [\log_a m + \log_a n = \log_a mn] \\ &= \log 40 - \log 4 \\ &= \log \frac{40}{4} && [\log_a m - \log_a n = \log_a \frac{m}{n}] \\ &= \log 10 \\ &= 1\end{aligned}$$

(ii) Consider the given expression

$$\begin{aligned}\log_{10} 8 + \log_{10} 25 + 2\log_{10} 3 - \log_{10} 18 \\ &= \log_{10} 8 + \log_{10} 25 + \log_{10} 3^2 - \log_{10} 18 \\ &&& [n \log_a m = \log_a m^n] \\ &= \log_{10} 8 + \log_{10} 25 + \log_{10} 9 - \log_{10} 18 \\ \\ &= \log_{10} 8 \times 25 \times 9 - \log_{10} 18 \\ &&& [\log_a l + \log_a m + \log_a n = \log_a lmn] \\ &= \log_{10} 1800 - \log_{10} 18 \\ &= \log_{10} \frac{1800}{18} && [\log_a m - \log_a n = \log_a \frac{m}{n}] \\ &= \log_{10} 100 \\ &= 2 && [\because \log_{10} 100 = 2]\end{aligned}$$

(iii) Consider the given expression

$$\begin{aligned}\log 4 + \frac{1}{3} \log 125 - \frac{1}{5} \log 32 \\ &= \log 4 + \log (125)^{\frac{1}{3}} - \log (32)^{\frac{1}{5}} && [n \log_a m = \log_a m^n] \\ &= \log 4 + \log (5^3)^{\frac{1}{3}} - \log (2^5)^{\frac{1}{5}} \\ &= \log 4 + \log 5 - \log 2 \\ &= \log 4 \times 5 - \log 2 && [\log_a m + \log_a n = \log_a mn] \\ &= \log \frac{20}{2} && [\log_a m - \log_a n = \log_a \frac{m}{n}] \\ &= \log 10 \\ &= 1\end{aligned}$$





## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 4:

We need to prove that

$$2\log \frac{15}{18} - \log \frac{25}{162} + \log \frac{4}{9} = \log 2$$

$$L.H.S = 2\log \frac{15}{18} - \log \frac{25}{162} + \log \frac{4}{9}$$

$$= \log \left( \frac{15}{18} \right)^2 - \log \frac{25}{162} + \log \frac{4}{9} \quad [n \log_a m = \log_a m^n]$$

$$= \log \left[ \left( \frac{15}{18} \right) \times \left( \frac{15}{18} \right) \right] - \log \frac{25}{162} + \log \frac{4}{9}$$

$$= \log \left( \frac{15}{18} \right) \times \left( \frac{15}{18} \right) \times \frac{4}{9} - \log \frac{25}{162} \quad [\log_a m + \log_a n = \log_a mn]$$

$$= \log \frac{\left( \frac{15}{18} \right) \times \left( \frac{15}{18} \right) \times \frac{4}{9}}{\frac{25}{162}} \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$

$$= \log \left( \frac{15}{18} \right) \times \left( \frac{15}{18} \right) \times \frac{4}{9} \times \frac{162}{25}$$

$$= \log \frac{72}{36}$$

$$= \log 2$$

$$= R.H.S$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai - 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 5:

Consider the given equation

$$x - \log 48 + 3\log 2 = \frac{1}{3}\log 125 - \log 3$$

$$\Rightarrow x = \frac{1}{3}\log 125 - \log 3 + \log 48 - 3\log 2$$

$$\Rightarrow x = \log(125)^{\frac{1}{3}} - \log 3 + \log 48 - \log 2^3 \quad [n\log_b m = \log_b m^n]$$

$$\Rightarrow x = \log(5 \times 5 \times 5)^{\frac{1}{3}} - \log 3 + \log 48 - \log 8$$

$$\Rightarrow x = \log(5^3)^{\frac{1}{3}} - \log 3 + \log 48 - \log 8$$

$$\Rightarrow x = \log 5 - \log 3 + \log 48 - \log 8$$

$$\Rightarrow x = \log 5 + \log 48 - \log 3 - \log 8$$

$$\Rightarrow x = (\log 5 + \log 48) - (\log 3 + \log 8)$$

$$\Rightarrow x = (\log 5 \times 48) - (\log 3 \times 8) \quad [\log_b m + \log_b n = \log_b mn]$$

$$\Rightarrow x = \log \frac{5 \times 48}{3 \times 8} \quad [\log_b m - \log_b n = \log_b \frac{m}{n}]$$

$$\Rightarrow x = \log \frac{5 \times 6 \times 8}{3 \times 8}$$

$$\Rightarrow x = \log 10$$

$$\Rightarrow x = 1$$

### Solution 6:

$$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10 \quad [ \because \log_{10} 10 = 1 ]$$

$$= \log_{10} 2 \times 10 \quad [\log_b m + \log_b n = \log_b mn]$$

$$= \log_{10} 20$$



## Solution 7:

(i)

$$\log_{10}(x - 10) = 1$$

$$\Rightarrow \log_{10}(x - 10) = \log_{10}10$$

$$\Rightarrow x - 10 = 10$$

$$\Rightarrow x = 10 + 10$$

$$\Rightarrow x = 20$$

(ii)

$$\log(x^2 - 21) = 2$$

$$\Rightarrow \log(x^2 - 21) = \log 100$$

$$\Rightarrow x^2 - 21 = 100$$

$$\Rightarrow x^2 - 21 - 100 = 0$$

$$\Rightarrow x^2 - 121 = 0$$

$$\Rightarrow x^2 = 121$$

$$\Rightarrow x = \pm\sqrt{121}$$

$$\Rightarrow x = \pm 11$$

(iii)

$$\log(x - 2) + \log(x + 2) = \log 5$$

$$\Rightarrow \log(x - 2)(x + 2) = \log 5 \quad [\log_a m + \log_a n = \log_a mn]$$

$$\Rightarrow \log(x^2 - 4) = \log 5$$

$$\Rightarrow x^2 - 4 = 5$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm\sqrt{9}$$

$$\Rightarrow x = \pm\sqrt{3^2}$$

$$\Rightarrow x = \pm 3$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(iv)

$$\log(x + 5) + \log(x - 5) = 4\log 2 + 2\log 3$$

$$\Rightarrow \log(x + 5)(x - 5) = 4\log 2 + 2\log 3 \quad [(\log_a m + \log_a n = \log_a mn)]$$

$$\Rightarrow \log(x^2 - 25) = \log 2^4 + \log 3^2 \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow \log(x^2 - 25) = \log 16 + \log 9$$

$$\Rightarrow \log(x^2 - 25) = \log 16 \times 9 \quad [(\log_a m + \log_a n = \log_a mn)]$$

$$\Rightarrow \log(x^2 - 25) = \log 144$$

$$\Rightarrow x^2 - 25 = 144$$

$$\Rightarrow x^2 = 144 + 25$$

$$\Rightarrow x^2 = 169$$

$$\Rightarrow x = \pm\sqrt{169}$$

$$\Rightarrow x = \pm\sqrt{13^2}$$

$$\Rightarrow x = \pm 13$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 8:

(i)

$$\frac{\log 81}{\log 27} = x$$

$$\Rightarrow x = \frac{\log 81}{\log 27}$$

$$\Rightarrow x = \frac{\log 3 \times 3 \times 3 \times 3}{\log 3 \times 3 \times 3}$$

$$\Rightarrow x = \frac{\log 3^4}{\log 3^3}$$

$$\Rightarrow x = \frac{4 \log 3}{3 \log 3} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{4}{3}$$

$$\Rightarrow x = 1\frac{1}{3}$$

(ii)

$$\frac{\log 128}{\log 32} = x$$

$$\Rightarrow x = \frac{\log 128}{\log 32}$$

$$\Rightarrow x = \frac{\log 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{\log 2 \times 2 \times 2 \times 2 \times 2}$$

$$\Rightarrow x = \frac{\log 2^7}{\log 2^5}$$

$$\Rightarrow x = \frac{7 \log 2}{5 \log 2} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{7}{5}$$

$$\Rightarrow x = 1.4$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(iii)

$$\frac{\log 64}{\log 8} = \log x$$

$$\Rightarrow \log x = \frac{\log 64}{\log 8}$$

$$\Rightarrow \log x = \frac{\log 2 \times 2 \times 2 \times 2 \times 2 \times 2}{\log 2 \times 2 \times 2}$$

$$\Rightarrow \log x = \frac{\log 2^6}{\log 2^3}$$

$$\Rightarrow \log x = \frac{6 \log 2}{3 \log 2} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow \log x = \frac{6}{3}$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10 \times 10$$

$$\Rightarrow x = 100$$

(iv)

$$\frac{\log 225}{\log 15} = \log x$$

$$\Rightarrow \log x = \frac{\log 225}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15 \times 15}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15^2}{\log 15}$$

$$\Rightarrow \log x = \frac{2 \log 15}{\log 15} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10 \times 10$$

$$\Rightarrow x = 100$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 9:

Given that

$$\log x = m + n;$$

$$\log y = m - n;$$

Consider the expression  $\log \frac{10x}{y^2}$  :

$$\log \frac{10x}{y^2} = \log 10x - \log y^2$$

$$= \log 10x - 2\log y \quad [n \log_a m = \log_a m^n]$$

$$= \log 10 + \log x - 2\log y \quad [\log_a m + \log_a n = \log_a mn]$$

$$= 1 + \log x - 2\log y$$

$$= 1 + m + n - 2(m - n)$$

$$= 1 + m + n - 2m + 2n$$

$$\Rightarrow \log \frac{10x}{y^2} = 1 - m + 3n$$



### Solution 10:

(i)

We have,

$$\log 1 = 0 \text{ and } \log 1000 = 3$$

$$\therefore \log 1 \times \log 1000 = 0 \times 3 = 0$$

Thus the statement,  $\log 1 \times \log 1000 = 0$  is true

(ii)

We know that

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\therefore \frac{\log x}{\log y} \neq \log x - \log y$$

Thus the statement,  $\frac{\log x}{\log y} = \log x - \log y$  is false

(iii)

Given that

$$\frac{\log 25}{\log 5} = \log x$$

$$\Rightarrow \frac{\log 5 \times 5}{\log 5} = \log x$$

$$\Rightarrow \frac{\log 5^2}{\log 5} = \log x$$

$$\Rightarrow \frac{2 \log 5}{\log 5} = \log x \quad [\log_a m^n = n \log_a m]$$

$$\Rightarrow 2 = \log_{10} x$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 100$$

Thus the statement,  $x = 2$  is false

(iv)

We know that

$$\log x + \log y = \log xy$$

$$\therefore \log x + \log y \neq \log x \times \log y$$

Thus the statement  $\log x + \log y = \log x \times \log y$  is false





## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 11:

Given that  $\log_{10} 2 = a$  and  $\log_{10} 3 = b$

(i)

$$\begin{aligned}\log 12 &= \log 2 \times 2 \times 3 \\ &= \log 2 \times 2 + \log 3 \quad [\log_e mn = \log_e m + \log_e n] \\ &= \log 2^2 + \log 3 \\ &= 2\log 2 + \log 3 \quad [n\log_e m = \log_e m^n] \\ &= 2a + b \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]\end{aligned}$$

(ii)

$$\begin{aligned}\log 2.25 &= \log \frac{225}{100} \\ &= \log \frac{25 \times 9}{25 \times 4} \\ &= \log \frac{9}{4} \\ &= \log \left(\frac{3}{2}\right)^2 \\ &= 2\log \left(\frac{3}{2}\right) \quad [n\log_e m = \log_e m^n] \\ &= 2(\log 3 - \log 2) \quad [\log_e m - \log_e n = \log_e \frac{m}{n}] \\ &= 2(b - a) \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b] \\ &= 2b - 2a\end{aligned}$$

(iii)

$$\begin{aligned}\log 2\frac{1}{4} &= \log \frac{9}{4} \\ &= \log \left(\frac{3}{2}\right)^2 \\ &= 2\log \left(\frac{3}{2}\right) \quad [n\log_e m = \log_e m^n] \\ &= 2(\log 3 - \log 2) \quad [\log_e m - \log_e n = \log_e \frac{m}{n}] \\ &= 2(b - a) \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b] \\ &= 2b - 2a\end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(iv)

$$\begin{aligned}\log 5.4 &= \log \frac{54}{10} \\ &= \log \left( \frac{2 \times 3 \times 3 \times 3}{10} \right) \\ &= \log(2 \times 3 \times 3 \times 3) - \log_{10} 10 \quad [\log_e \frac{m}{n} = \log_e m - \log_e n] \\ &= \log_{10} 2 + \log_{10} 3^3 - \log_{10} 10 \quad [\log_e mn = \log_e m + \log_e n] \\ &= \log_{10} 2 + 3\log_{10} 3 - \log_{10} 10 \quad [n \log_e m = \log_e m^n] \\ &= \log_{10} 2 + 3\log_{10} 3 - 1 \quad [\because \log_{10} 10 = 1] \\ &= a + 3b - 1 \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]\end{aligned}$$

(v)

$$\begin{aligned}\log 60 &= \log_{10} 10 \times 2 \times 3 \\ &= \log_{10} 10 + \log_{10} 2 + \log_{10} 3 \quad [\log_e mn = \log_e m + \log_e n] \\ &= 1 + \log_{10} 2 + \log_{10} 3 \quad [\because \log_{10} 10 = 1] \\ &= 1 + a + b \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]\end{aligned}$$

(vi)

$$\begin{aligned}\log 3\frac{1}{8} &= \log_{10} \left( \frac{25}{8} \times \frac{4}{4} \right) \\ &= \log_{10} \left( \frac{100}{32} \right) \\ &= \log_{10} 100 - \log_{10} 32 \quad [\log_e \frac{m}{n} = \log_e m - \log_e n] \\ &= \log_{10} 100 - \log_{10} 2^5 \\ &= 2 - \log_{10} 2^5 \quad [\because \log_{10} 100 = 2] \\ &= 2 - 5\log_{10} 2 \quad [\log_e m^n = n \log_e m] \\ &= 2 - 5a \quad [\because \log_{10} 2 = a]\end{aligned}$$



## Solution 12:

We know that  $\log 2 = 0.3010$  and  $\log 3 = 0.4771$

(i)

$$\begin{aligned}\log 12 &= \log 2 \times 2 \times 3 \\ &= \log 2 \times 2 + \log 3 && [\log_a mn = \log_a m + \log_a n] \\ &= \log 2^2 + \log 3 \\ &= 2 \log 2 + \log 3 && [n \log_a m = \log_a m^n] \\ &= 2(0.3010) + 0.4771 && \left[ \begin{array}{l} \because \log 2 = 0.3010 \text{ and} \\ \log 3 = 0.4771 \end{array} \right] \\ &= 1.0791\end{aligned}$$

(ii)

$$\begin{aligned}\log 1.2 &= \log \frac{12}{10} \\ &= \log 12 - \log 10 && [\log_a \frac{m}{n} = \log_a m - \log_a n] \\ &= \log 2 \times 2 \times 3 - 1 && [\because \log 10 = 1] \\ &= \log 2 \times 2 + \log 3 - 1 && [\log_a mn = \log_a m + \log_a n] \\ &= \log 2^2 + \log 3 - 1 \\ &= 2 \log 2 + \log 3 - 1 && [n \log_a m = \log_a m^n] \\ &= 2(0.3010) + 0.4771 - 1 && \left[ \begin{array}{l} \because \log 2 = 0.3010 \\ \text{and } \log 3 = 0.4771 \end{array} \right] \\ &= 1.0791 - 1 \\ &= 0.0791\end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai - 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(iii)

$$\begin{aligned}\log 3.6 &= \log \frac{36}{10} \\ &= \log 36 - \log 10 && [\log_e \frac{m}{n} = \log_e m - \log_e n] \\ &= \log 2 \times 2 \times 3 \times 3 - 1 && [\because \log 10 = 1] \\ &= \log 2 \times 2 + \log 3 \times 3 - 1 && [\log_e mn = \log_e m + \log_e n] \\ &= \log 2^2 + \log 3^2 - 1 \\ &= 2 \log 2 + 2 \log 3 - 1 && [n \log_e m = \log_e m^n] \\ &= 2(0.3010) + 2(0.4771) - 1 && [\because \log 2 = 0.3010 \\ &&& \text{and } \log 3 = 0.4771] \\ &= 1.5562 - 1 \\ &= 0.5562\end{aligned}$$

(iv)

$$\begin{aligned}\log 15 &= \log \left( \frac{15}{10} \times 10 \right) \\ &= \log \left( \frac{15}{10} \right) + \log 10 \\ &= \log \left( \frac{3}{2} \right) + 1 && [\because \log 10 = 1] \\ &= \log 3 - \log 2 + 1 && [\because \log m - \log n = \log \left( \frac{m}{n} \right)] \\ &= 0.4771 - 0.3010 + 1 \\ &= 1.1761\end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai - 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(v)

$$\begin{aligned}\log 25 &= \log\left(\frac{25}{4} \times 4\right) \\ &= \log\left(\frac{100}{4}\right) && [\log_e mn = \log_e m + \log_e n] \\ &= \log 100 - \log(2 \times 2) && [\log_e \frac{m}{n} = \log_e m - \log_e n] \\ &= 2 - \log(2^2) && [\log 100 = 2] \\ &= 2 - 2\log 2 && [\log_e m^n = n \log_e m] \\ &= 2 - 2(0.3010) && [\therefore \log 2 = 0.3010] \\ &= 1.398\end{aligned}$$

(vi)

$$\begin{aligned}\frac{2}{3}\log 8 &= \frac{2}{3}\log 2 \times 2 \times 2 \\ &= \frac{2}{3}\log 2^3 \\ &= 3 \times \frac{2}{3}\log 2 && [\log_e m^n = n \log_e m] \\ &= 2\log 2 \\ &= 2 \times 0.3010 && [\therefore \log 2 = 0.3010] \\ &= 0.602\end{aligned}$$



### Solution 13:

(i)

Consider the given equation:

$$2\log_{10} x + 1 = \log_{10} 250$$

$$\Rightarrow \log_{10} x^2 + 1 = \log_{10} 250 \quad [\log_a m^n = n\log_a m]$$

$$\Rightarrow \log_{10} x^2 + \log_{10} 10 = \log_{10} 250 \quad [\because \log_{10} 10 = 1]$$

$$\Rightarrow \log_{10} (x^2 \times 10) = \log_{10} 250 \quad [\log_a m + \log_a n = \log_a mn]$$

$$\Rightarrow x^2 \times 10 = 250$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \sqrt{25}$$

$$\Rightarrow x = 5$$

(ii)

$x = 5$  (proved above in (i))

$$\log_{10} 2x = \log_{10} 2(5)$$

$$= \log_{10} 10$$

$$= 1 \quad [\because \log_{10} 10 = 1]$$

### Solution 14:

$$3\log x + \frac{1}{2}\log y = 2$$

$$\Rightarrow \log x^3 + \log \sqrt{y} = 2$$

$$\Rightarrow \log x^3 \sqrt{y} = 2$$

$$\Rightarrow x^3 \sqrt{y} = 10^2$$

$$\Rightarrow \sqrt{y} = \frac{10^2}{x^3}$$

Squaring both sides, we get

$$y = \frac{10000}{x^6}$$

$$\Rightarrow y = 10000x^{-6}$$



### Solution 15:

$$x = (100)^a, y = (10000)^b \text{ and } z = (10)^c$$

$$\Rightarrow \log x = a \log 100, \log y = b \log 10000 \text{ and } \log z = c \log 10$$

$$\begin{aligned} \log \frac{10\sqrt{y}}{x^2z^3} &= \log 10\sqrt{y} - \log(x^2z^3) \\ &= \log(10y^{1/2}) - \log x^2 - \log z^3 \\ &= \log 10 + \log y^{1/2} - \log x^2 - \log z^3 \\ &= \log 10 + \frac{1}{2} \log y - 2 \log x - 3 \log z \\ &= 1 + \frac{1}{2} \log(10000)^b - 2 \log(100)^a - 3 \log(10)^c \dots\dots (\text{Since } \log 10 = 1) \\ &= 1 + \frac{b}{2} \log(10)^4 - a \log(10)^2 - 3c \log 10 \\ &= 1 + \frac{b}{2} \times 4 \log 10 - 2 \times 2a \log 10 - 3c \log 10 \\ &= 1 + 2b - 4a - 3c \end{aligned}$$

### Solution 16:

$$3(\log 5 - \log 3) - (\log 5 - 2 \log 6) = 2 - \log x$$

$$\begin{aligned} \Rightarrow 3 \log 5 - 3 \log 3 - \log 5 + 2 \log(2 \times 3) &= 2 - \log x \\ \Rightarrow 3 \log 5 - 3 \log 3 - \log 5 + 2 \log 2 + 2 \log 3 &= 2 - \log x \\ \Rightarrow 2 \log 5 - \log 3 + 2 \log 2 &= 2 - \log x \\ \Rightarrow 2 \log 5 - \log 3 + 2 \log 2 + \log x &= 2 \\ \Rightarrow \log 5^2 - \log 3 + \log 2^2 + \log x &= 2 \\ \Rightarrow \log \left( \frac{25 \times 4 \times x}{3} \right) &= 2 \\ \Rightarrow \log \left( \frac{100x}{3} \right) &= 2 \\ \Rightarrow \frac{100x}{3} &= 10^2 \\ \Rightarrow \frac{x}{3} &= 1 \\ \Rightarrow x &= 3 \end{aligned}$$

### Exercise 8(C)



## Solution 1:

$$\text{Given that } \log_{10} 8 = 0.90$$

$$\Rightarrow \log_{10} 2 \times 2 \times 2 = 0.90$$

$$\Rightarrow \log_{10} 2^3 = 0.90$$

$$\Rightarrow 3\log_{10} 2 = 0.90$$

$$\Rightarrow \log_{10} 2 = \frac{0.90}{3}$$

$$\Rightarrow \log_{10} 2 = 0.30 \dots (1)$$

(i)

$$\log 4 = \log_{10} (2 \times 2)$$

$$\Rightarrow = \log_{10} (2^2)$$

$$\Rightarrow = 2\log_{10} 2$$

$$\Rightarrow = 2(0.30) \quad [\text{from (1)}]$$

$$\Rightarrow = 0.60$$

(ii)

$$\log \sqrt{32} = \log_{10} (32)^{\frac{1}{2}}$$

$$\Rightarrow = \frac{1}{2} \log_{10} (32)$$

$$\Rightarrow = \frac{1}{2} \log_{10} (2 \times 2 \times 2 \times 2 \times 2)$$

$$\Rightarrow = \frac{1}{2} \log_{10} (2^5)$$

$$\Rightarrow = \frac{1}{2} \times 5 \log_{10} 2$$

$$\Rightarrow = \frac{1}{2} \times 5(0.30) \quad [\text{from (1)}]$$

$$\Rightarrow = 5 \times 0.15$$

$$\Rightarrow = 0.75$$





## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(iii)

$$\begin{aligned}\log 0.125 &= \log_{10} \frac{125}{1000} \\ &= \log_{10} \frac{1}{8} \\ &= \log_{10} \frac{1}{2 \times 2 \times 2} \\ &= \log_{10} \left( \frac{1}{2^3} \right) \\ &= \log_{10} 2^{-3} \\ &= -3 \times (0.30) \quad [\text{from (1)}] \\ &= -0.9\end{aligned}$$

**Solution 2:**

$$\begin{aligned}\log 27 &= 1.431 \\ \Rightarrow \log 3 \times 3 \times 3 &= 1.431 \\ \Rightarrow \log 3^3 &= 1.431 \\ \Rightarrow 3 \log 3 &= 1.431 \\ \Rightarrow \log 3 &= \frac{1.431}{3} \\ \Rightarrow \log 3 &= 0.477 \dots (1)\end{aligned}$$

(i)

$$\begin{aligned}\log 9 &= \log(3 \times 3) \\ &= \log 3^2 \\ &= 2 \log 3 \\ &= 2 \times 0.477 \quad [\text{from (1)}] \\ &= 0.954\end{aligned}$$

(ii)

$$\begin{aligned}\log 300 &= \log(3 \times 100) \\ &= \log 3 + \log 100 \\ &= \log 3 + 2 \quad [ \because \log_{10} 100 = 2 ] \\ &= 0.477 + 2 \quad [\text{from (1)}] \\ &= 2.477\end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 3:

$$\log_{10} a = b$$

$$\Rightarrow 10^b = a$$

$$\Rightarrow (10^b)^3 = (a)^3 \quad [\text{cubing both sides}]$$

$$\Rightarrow \frac{10^{3b}}{10^2} = \frac{a^3}{10^2} \quad [\text{dividing both sides by } 10^2]$$

$$\Rightarrow 10^{3b-2} = \frac{a^3}{100}$$

### Solution 4:

$$\log_5 x = y \quad [\text{given}]$$

$$\Rightarrow 5^y = x$$

$$\Rightarrow (5^y)^2 = x^2$$

$$\Rightarrow 5^{2y} = x^2$$

$$\Rightarrow 5^{2y} \times 5^3 = x^2 \times 5^3$$

$$\Rightarrow 5^{2y+3} = 125x^2$$



## Solution 5:

Given that  $\log_3 m = x$  and  $\log_3 n = y$

$$\Rightarrow 3^x = m \text{ and } 3^y = n$$

(i)

Consider the given expression:

$$3^{2x-3} = 3^{2x} \cdot 3^{-3}$$

$$= 3^{2x} \cdot \frac{1}{3^3}$$

$$= \frac{3^{2x}}{3^3}$$

$$= \frac{(3^x)^2}{3^3}$$

$$= \frac{m^2}{27}$$

$$\text{Therefore, } 3^{2x-3} = \frac{m^2}{27}$$

(ii)

Consider the given expression:

$$3^{1-2y+3x} = 3^1 \cdot 3^{-2y} \cdot 3^{3x}$$

$$= 3 \cdot \frac{1}{3^{2y}} \cdot 3^{3x}$$

$$= \frac{3}{(3^y)^2} \cdot (3^x)^3$$

$$= \frac{3}{(n)^2} \cdot (m)^3$$

$$= \frac{3m^3}{n^2}$$

$$\text{Therefore, } 3^{1-2y+3x} = \frac{3m^3}{n^2}$$



(iii)

Consider the given expression:

$$2 \log_3 A = 5x - 3y$$

$$\Rightarrow 2 \log_3 A = 5 \log_3 m - 3 \log_3 n$$

$$\Rightarrow \log_3 A^2 = \log_3 m^5 - \log_3 n^3$$

$$\Rightarrow \log_3 A^2 = \log_3 \left( \frac{m^5}{n^3} \right)$$

$$\Rightarrow A^2 = \left( \frac{m^5}{n^3} \right)$$

$$\Rightarrow A = \sqrt{\left( \frac{m^5}{n^3} \right)}$$

**Solution 6:**

(i)

$$\begin{aligned} \log(a)^3 - \log a &= 3 \log a - \log a \\ &= 2 \log a \end{aligned}$$

(ii)

$$\begin{aligned} \log(a)^3 + \log a &= 3 \log a + \log a \\ &= \frac{3 \log a}{\log a} \\ &= 3 \end{aligned}$$

**Solution 7:**

$$\log(a + b) = \log a + \log b$$

$$\Rightarrow \log(a + b) = \log ab$$

$$\Rightarrow a + b = ab$$

$$\Rightarrow a - ab = -b$$

$$\Rightarrow -ab + a = -b$$

$$\Rightarrow -a(b - 1) = -b$$

$$\Rightarrow a(b - 1) = b$$

$$\Rightarrow a = \frac{b}{b - 1}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 8:

(i)

$$\begin{aligned}L.H.S &= (\log a)^2 - (\log b)^2 \\ \Rightarrow L.H.S &= (\log a + \log b)(\log a - \log b) \\ \Rightarrow L.H.S &= \log(ab) \log\left(\frac{a}{b}\right) \\ \Rightarrow L.H.S &= \log\left(\frac{a}{b}\right) \times \log(ab) \\ \Rightarrow L.H.S &= R.H.S\end{aligned}$$

Hence proved.

(ii)

Given that

$$\begin{aligned}a \log b + b \log a - 1 &= 0 \\ \Rightarrow a \log b + b \log a &= 1 \\ \Rightarrow \log b^a + \log a^b &= 1 \\ \Rightarrow \log b^a + \log a^b &= \log 10 \\ \Rightarrow \log(b^a \cdot a^b) &= \log 10 \\ \Rightarrow b^a \cdot a^b &= 10\end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 9:

(i)

Given that

$$\log(a+1) = \log(4a-3) - \log 3$$

$$\Rightarrow \log(a+1) = \log\left(\frac{4a-3}{3}\right)$$

$$\Rightarrow a+1 = \frac{4a-3}{3}$$

$$\Rightarrow 3a+3 = 4a-3$$

$$\Rightarrow 4a-3a = 3+3$$

$$\Rightarrow a = 6$$

(ii)

$$2\log y - \log x - 3 = 0$$

$$\Rightarrow 2\log y - \log x = 3$$

$$\Rightarrow \log y^2 - \log x = 3$$

$$\Rightarrow \log y^2 - \log x = \log 1000$$

$$\Rightarrow \log \frac{y^2}{x} = \log 1000$$

$$\Rightarrow \frac{y^2}{x} = 1000$$

$$\Rightarrow x = \frac{y^2}{1000}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

(iii)

$$\log_{10} 125 = 3(1 - \log_{10} 2)$$

$$L.H.S. = \log_{10} 125$$

$$= \log_{10} 5 \times 5 \times 5$$

$$= \log_{10} 5^3$$

$$= 3\log_{10} 5 \dots (1)$$

$$R.H.S. = 3(1 - \log_{10} 2)$$

$$= 3(\log_{10} 10 - \log_{10} 2)$$

$$= 3\log_{10} \left(\frac{10}{2}\right)$$

$$= 3\log_{10} 5 \dots (2)$$

From (1) and (2), we have

$$L.H.S. = R.H.S.$$

Hence proved.

### Solution 10:

Given  $\log x = 2m - n$ ,  $\log y = n - 2m$  and  $\log z = 3m - 2n$

$$\begin{aligned} \log \frac{x^2 y^3}{z^4} &= \log x^2 y^3 - \log z^4 \\ &= \log x^2 + \log y^3 - \log z^4 \\ &= 2\log x + 3\log y - 4\log z \\ &= 2(2m - n) + 3(n - 2m) - 4(3m - 2n) \\ &= 4m - 2n + 3n - 6m - 12m + 8n \\ &= -14m + 7n \end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 11:

$$\log_x 25 - \log_x 5 = 2 - \log_x \frac{1}{125}$$

$$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x \left(\frac{1}{5}\right)^3$$

$$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x 5^{-3}$$

$$\Rightarrow 2\log_x 5 - \log_x 5 = 2 + 3\log_x 5$$

$$\Rightarrow 2\log_x 5 - \log_x 5 - 3\log_x 5 = 2$$

$$\Rightarrow -2\log_x 5 = 2$$

$$\Rightarrow \log_x 5 = -1$$

$$\Rightarrow x^{-1} = 5$$

$$\Rightarrow \frac{1}{x} = 5$$

$$\Rightarrow x = \frac{1}{5}$$

### Exercise 8(D)

#### Solution 1:

$$\frac{3}{2}\log a + \frac{2}{3}\log b - 1 = 0$$

$$\Rightarrow \log a^{\frac{3}{2}} + \log b^{\frac{2}{3}} = 1$$

$$\Rightarrow \log \left( a^{\frac{3}{2}} \times b^{\frac{2}{3}} \right) = 1$$

$$\Rightarrow \log \left( a^{\frac{3}{2}} \times b^{\frac{2}{3}} \right) = \log 10$$

$$\Rightarrow a^{\frac{3}{2}} \times b^{\frac{2}{3}} = 10$$

$$\Rightarrow \left( a^{\frac{3}{2}} \times b^{\frac{2}{3}} \right)^6 = 10^6$$

$$\Rightarrow a^9 \cdot b^4 = 10^6$$





## Solution 2:

Given that

$$x = 1 + \log 2 - \log 5, y = 2 \log 3 \text{ and } z = \log a - \log 5$$

Consider

$$\begin{aligned}x &= 1 + \log 2 - \log 5 \\&= \log 10 + \log 2 - \log 5 \\&= \log(10 \times 2) - \log 5 \\&= \log 20 - \log 5 \\&= \log \frac{20}{5}\end{aligned}$$

$$= \log 4 \dots (1)$$

We have

$$\begin{aligned}y &= 2 \log 3 \\&= \log 3^2 \\&= \log 9 \dots (2)\end{aligned}$$

Also we have

$$\begin{aligned}z &= \log a - \log 5 \\&= \log \frac{a}{5} \dots (3)\end{aligned}$$

Given that  $x + y = 2z$

$\therefore$  Substitute the values of  $x, y$  and  $z$  from (1), (2) and (3), we have

$$\begin{aligned}\Rightarrow \log 4 + \log 9 &= 2 \log \frac{a}{5} \\ \Rightarrow \log 4 + \log 9 &= \log \left( \frac{a}{5} \right)^2 \\ \Rightarrow \log 4 + \log 9 &= \log \frac{a^2}{25} \\ \Rightarrow \log(4 \times 9) &= \log \frac{a^2}{25} \\ \Rightarrow \log 36 &= \log \frac{a^2}{25} \\ \Rightarrow \frac{a^2}{25} &= 36 \\ \Rightarrow a^2 &= 36 \times 25 \\ \Rightarrow a^2 &= 900 \\ \Rightarrow a &= 30\end{aligned}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 3:

Given that

$$x = \log 0.6, y = \log 1.25, z = \log 3 - 2 \log 2$$

Consider

$$z = \log 3 - 2 \log 2$$

$$= \log 3 - \log 2^2$$

$$= \log 3 - \log 4$$

$$= \log \frac{3}{4}$$

$$= \log 0.75 \dots (1)$$

(i)

$$x + y - z = \log 0.6 + \log 1.25 - \log 0.75$$

$$= \log \frac{0.6 \times 1.25}{0.75}$$

$$= \log \frac{0.75}{0.75}$$

$$= \log 1$$

$$= 0 \dots (2)$$

(ii)

$$5^{x+y-z} = 5^0 \dots [\because x + y - z = 0 \text{ from (2)}]$$

$$= 1$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 4:

Given that

$$a^2 = \log x, b^3 = \log y \text{ and } 3a^2 - 2b^3 = 6 \log z$$

Consider the equation,

$$3a^2 - 2b^3 = 6 \log z$$

$$\Rightarrow 3 \log x - 2 \log y = 6 \log z$$

$$\Rightarrow \log x^3 - \log y^2 = \log z^6$$

$$\Rightarrow \log \left( \frac{x^3}{y^2} \right) = \log z^6$$

$$\Rightarrow \frac{x^3}{y^2} = z^6$$

$$\Rightarrow \frac{x^3}{z^6} = y^2$$

$$\Rightarrow y^2 = \frac{x^3}{z^6}$$

$$\Rightarrow y = \left( \frac{x^3}{z^6} \right)^{\frac{1}{2}}$$

$$\Rightarrow y = \left( \frac{x^{\frac{3}{2}}}{z^{\frac{6}{2}}} \right)$$

$$\Rightarrow y = \frac{x^{\frac{3}{2}}}{z^3}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 5:

$$\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$$

$$\Rightarrow \log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log ab)$$

$$\Rightarrow \log\left(\frac{a-b}{2}\right) = \log(ab)^{\frac{1}{2}}$$

$$\Rightarrow \left(\frac{a-b}{2}\right) = (ab)^{\frac{1}{2}}$$

Squaring both sides we have,

$$\left(\frac{a-b}{2}\right)^2 = ab$$

$$\Rightarrow \frac{(a-b)^2}{4} = ab$$

$$\Rightarrow (a-b)^2 = 4ab$$

$$\Rightarrow a^2 + b^2 - 2ab = 4ab$$

$$\Rightarrow a^2 + b^2 = 4ab + 2ab$$

$$\Rightarrow a^2 + b^2 = 6ab$$



## Solution 6:

Given that

$$a^2 + b^2 = 23ab$$

$$\Rightarrow a^2 + b^2 + 2ab = 23ab + 2ab$$

$$\Rightarrow a^2 + b^2 + 2ab = 25ab$$

$$\Rightarrow (a+b)^2 = 25ab$$

$$\Rightarrow \frac{(a+b)^2}{25} = ab$$

$$\Rightarrow \left(\frac{a+b}{5}\right)^2 = ab$$

$$\Rightarrow \log\left(\frac{a+b}{5}\right)^2 = \log ab$$

$$\Rightarrow 2\log\left(\frac{a+b}{5}\right) = \log ab$$

$$\Rightarrow \log\left(\frac{a+b}{5}\right) = \frac{1}{2}(\log a + \log b)$$

## Solution 7:

Given that

$$m = \log 20 \text{ and } n = \log 25$$

We also have

$$2\log(x-4) = 2m - n$$

$$\Rightarrow 2\log(x-4) = 2\log 20 - \log 25$$

$$\Rightarrow \log(x-4)^2 = \log 20^2 - \log 25$$

$$\Rightarrow \log(x-4)^2 = \log 400 - \log 25$$

$$\Rightarrow \log(x-4)^2 = \log \frac{400}{25}$$

$$\Rightarrow (x-4)^2 = \frac{400}{25}$$

$$\Rightarrow (x-4)^2 = 16$$

$$\Rightarrow x-4 = 4$$

$$\Rightarrow x = 4 + 4$$

$$\Rightarrow x = 8$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 8:

v

$$\Rightarrow 4x = 100y$$

$$\Rightarrow x = 25y$$

$$\Rightarrow xy = 25y \times y$$

$$\Rightarrow xy = 25y^2$$

$$\Rightarrow 100 = 25y^2 \dots \text{[from(1)]}$$

$$\Rightarrow y^2 = \frac{100}{25}$$

$$\Rightarrow y^2 = 4$$

$$\Rightarrow y = 2 \text{ } [\because y > 0]$$

From (1),

$$xy = 100$$

$$\Rightarrow x \times 2 = 100$$

$$\Rightarrow x = \frac{100}{2}$$

$$\Rightarrow x = 50$$

Thus the values of x and y are x=50 and y=2



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 9:

(i)

$$\log_x 625 = 4$$

$$\Rightarrow 625 = x^{-4} \text{ [Removing Logarithm]}$$

$$\Rightarrow 5^4 = \left(\frac{1}{x}\right)^4$$

$$\Rightarrow 5 = \frac{1}{x} \text{ [Powers are same, bases are equal]}$$

$$\Rightarrow x = \frac{1}{5}$$

(ii)

$$\log_x(5x-6) = 2$$

$$\Rightarrow 5x-6 = x^2 \text{ [Removing Logarithm]}$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow x(x-3) - 2(x-3) = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\therefore x = 2, 3$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 10:

Given that

$$p = \log 20 \text{ and } q = \log 25$$

we also have

$$2\log(x + 1) = 2p - q$$

$$\Rightarrow 2\log(x + 1) = 2\log 20 - \log 25$$

$$\Rightarrow \log(x + 1)^2 = \log 20^2 - \log 25$$

$$\Rightarrow \log(x + 1)^2 = \log 400 - \log 25$$

$$\Rightarrow \log(x + 1)^2 = \log \frac{400}{25}$$

$$\Rightarrow \log(x + 1)^2 = \log 16$$

$$\Rightarrow \log(x + 1)^2 = \log 4^2$$

$$\Rightarrow x + 1 = 4$$

$$\Rightarrow x = 4 - 1$$

$$\Rightarrow x = 3$$





## Solution 11:

$$\log_2(x + y) = \frac{\log 25}{\log 0.2}$$

$$\Rightarrow \log_2(x + y) = \log_{0.2} 25$$

$$\Rightarrow \log_2(x + y) = \log_{\frac{1}{10}} 25$$

$$\Rightarrow \log_2(x + y) = \log_{5^{-1}} 5^2$$

$$\Rightarrow \log_2(x + y) = -2\log_5 5$$

$$\Rightarrow \log_2(x + y) = -2$$

$$\Rightarrow x + y = 2^{-2} \text{ [Removing logarithm]}$$

$$\Rightarrow x + y = \frac{1}{4} \dots\dots(i)$$

$$\log_3(x - y) = \frac{\log 25}{\log 0.2}$$

$$\Rightarrow \log_3(x - y) = \log_{0.2} 25$$

$$\Rightarrow \log_3(x - y) = \log_{\frac{1}{10}} 25$$

$$\Rightarrow \log_3(x - y) = \log_{5^{-1}} 5^2$$

$$\Rightarrow \log_3(x - y) = -2\log_5 5$$

$$\Rightarrow \log_3(x - y) = -2$$

$$\Rightarrow x - y = 3^{-2} \text{ [Removing logarithm]}$$

$$\Rightarrow x - y = \frac{1}{9} \dots\dots(ii)$$

Solving (i) & (ii), we get

$$x = \frac{13}{72}, y = \frac{5}{72}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 12:

$$\frac{\log x}{\log y} = \frac{3}{2}$$

$$\Rightarrow 2\log x = 3\log y$$

$$\Rightarrow \log y = \frac{2\log x}{3} \dots\dots(i)$$

$$\log(xy) = 5$$

$$\Rightarrow \log x + \log y = 5$$

$$\Rightarrow \log x + \frac{2\log x}{3} = 5 \text{ [Substituting (i)]}$$

$$\Rightarrow \frac{3\log x + 2\log x}{3} = 5$$

$$\Rightarrow \frac{5\log x}{3} = 5$$

$$\Rightarrow \log x = 3$$

$$\Rightarrow x = 10^3$$

$$\therefore x = 1000$$

Substituting  $x = 1000$

$$\log y = \frac{2 \times 3}{3}$$

$$\Rightarrow \log y = 2$$

$$\Rightarrow y = 10^2$$

$$\therefore y = 100$$



### Solution 13:

$$(i) \log_{10} x = 2a$$

$$\Rightarrow x = 10^{2a} \text{ [Removing logarithm from both sides]}$$

$$\Rightarrow x^{1/2} = 10^a$$

$$\Rightarrow 10^a = x^{1/2}$$

$$(ii) \log_{10} y = \frac{b}{2}$$

$$\Rightarrow y = 10^{b/2}$$

$$\Rightarrow y^4 = 10^{2b}$$

$$\Rightarrow 10y^4 = 10^{2b} \times 10$$

$$\Rightarrow 10^{2b+1} = 10y^4$$

(iii)

$$\text{We know } 10^a = x^{1/2}$$

$$10^{b/2} = y$$

$$\Rightarrow 10^b = y^2$$

$$\log_{10} p = 3a - 2b$$

$$\Rightarrow p = 10^{3a-2b}$$

$$\Rightarrow p = (10^3)^a + (10^2)^b$$

$$\Rightarrow p = (10^a)^3 + (10^b)^2$$

Substituting  $10^a$  &  $10^b$ , we get

$$\Rightarrow p = (x^{1/2})^3 + (y^2)^2$$

$$\Rightarrow p = x^{3/2} + y^4$$

$$\Rightarrow p = \frac{x^{3/2}}{y^4}$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 14:

$$\log_5(x + 1) - 1 = 1 + \log_5(x - 1)$$

$$\Rightarrow \log_5(x + 1) - \log_5(x - 1) = 2$$

$$\Rightarrow \log_5 \frac{(x+1)}{(x-1)} = 2$$

$$\Rightarrow \frac{(x + 1)}{(x - 1)} = 5^2$$

$$\Rightarrow \frac{(x + 1)}{(x - 1)} = 25$$

$$\Rightarrow x + 1 = 25(x - 1)$$

$$\Rightarrow x + 1 = 25x - 25$$

$$\Rightarrow 25x - x = 25 + 1$$

$$\Rightarrow 24x = 26$$

$$\Rightarrow x = \frac{26}{24} = \frac{13}{12}$$

### Solution 15:

$$\log_x 49 - \log_x 7 + \log_x \frac{1}{343} = -2$$

$$\Rightarrow \log_x \frac{49}{7 \times 343} = -2$$

$$\Rightarrow \log_x \frac{1}{49} = -2$$

$$\Rightarrow -\log_x 49 = -2$$

$$\Rightarrow \log_x 49 = 2$$

$$\Rightarrow 49 = x^2 \text{ [Removing logarithm]}$$

$$\therefore x = 7$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 16:

$$\text{Given } a^2 = \log x, b^3 = \log y$$

$$\text{Now } \frac{a^2}{2} - \frac{b^3}{3} = \log c$$

$$\Rightarrow \frac{\log x}{2} - \frac{\log y}{3} = \log c$$

$$\Rightarrow \frac{3\log x - 2\log y}{6} = \log c$$

$$\Rightarrow 3\log x - 2\log y = 6\log c$$

$$\Rightarrow \log x^3 - \log y^2 = 6\log c$$

$$\Rightarrow \log \left( \frac{x^3}{y^2} \right) = \log c^6$$

$$\Rightarrow \frac{x^3}{y^2} = c^6$$

$$\Rightarrow c = \sqrt[6]{\frac{x^3}{y^2}}$$

### Solution 17:

$$x - y - z$$

$$= \log_{10} 12 - \log_4 2 \times \log_{10} 9 - \log_{10} 0.4$$

$$= \log_{10} (4 \times 3) - \log_4 2 \times \log_{10} 9 - \log_{10} 0.4$$

$$= \log_{10} 4 + \log_{10} 3 - \log_4 2 \times 2\log_{10} 3 - \log_{10} \left( \frac{4}{10} \right)$$

$$= \log_{10} 4 + \log_{10} 3 - \frac{\log_{10} 2}{2\log_{10} 2} \times 2\log_{10} 3 - \log_{10} 4 + \log_{10} 10$$

$$= \log_{10} 4 + \log_{10} 3 - \frac{2\log_{10} 3}{2} - \log_{10} 4 + 1$$

$$= 1$$

$$\text{(ii) } 13^{x-y-z} = 13^1 = 13$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 18:

$$\begin{aligned}\log_x 15\sqrt{5} &= 2 - \log_x 3\sqrt{5} \\ \Rightarrow \log_x 15\sqrt{5} + \log_x 3\sqrt{5} &= 2 \\ \Rightarrow \log_x (15\sqrt{5} \times 3\sqrt{5}) &= 2 \\ \Rightarrow \log_x 225 &= 2 \\ \Rightarrow \log_x 15^2 &= 2 \\ \Rightarrow 2\log_x 15 &= 2 \\ \Rightarrow \log_x 15 &= 1 \\ \Rightarrow x &= 15\end{aligned}$$



## Solution 19:

$$(i) \log_b a \times \log_c b \times \log_a c$$

$$= \frac{\log_{10} a}{\log_{10} b} \times \frac{\log_{10} b}{\log_{10} c} \times \frac{\log_{10} c}{\log_{10} a}$$

$$= 1$$

$$(ii) \log_3 8 \div \log_9 16$$

$$= \frac{\log_3 8}{\log_9 16}$$

$$= \frac{\log_{10} 8}{\log_{10} 3} \times \frac{\log_{10} 9}{\log_{10} 16}$$

$$= \frac{3\log_{10} 2}{\log_{10} 3} \times \frac{2\log_{10} 3}{4\log_{10} 2}$$

$$= \frac{3}{2}$$

$$(iii) \frac{\log_5 8}{\log_{25} 16 \times \log_{100} 10}$$

$$= \frac{\frac{\log_{10} 8}{\log_{10} 5}}{\frac{\log_{10} 16}{\log_{10} 25} \times \frac{\log_{10} 10}{\log_{10} 100}}$$

$$= \frac{\frac{\log_{10} 2^3}{\log_{10} 5}}{\frac{\log_{10} 2^4}{\log_{10} 5^2} \times \frac{\log_{10} 10}{\log_{10} 10^2}}$$

$$= \frac{\log_{10} 2^3}{\log_{10} 5} \times \frac{\log_{10} 5^2}{\log_{10} 2^4} \times \frac{\log_{10} 10^2}{\log_{10} 10}$$

$$= \frac{3\log_{10} 2}{\log_{10} 5} \times \frac{2\log_{10} 5}{4\log_{10} 2} \times \frac{2\log_{10} 10}{\log_{10} 10}$$

$$= 3$$



## Student's Favourite Academy

Shop No. 5, "Umang" Vasant Utsav C H S Ltd., Thakur Village, Kandivali E, Mumbai – 400 101 Phone : 8828132765, 9833035468  
Email : favouriteacademy@gmail.com

### Solution 20:

$$\begin{aligned}\log_a m \div \log_{ab} m &= \frac{\log_a m}{\log_{ab} m} \\ &= \frac{\log_m ab}{\log_m a} \left[ Q \log_b a = \frac{1}{\log_a b} \right] \\ &= \log_a ab \left[ Q \frac{\log_x a}{\log_x b} = \log_b a \right] \\ &= \log_a a + \log_a b \\ &= 1 + \log_a b\end{aligned}$$